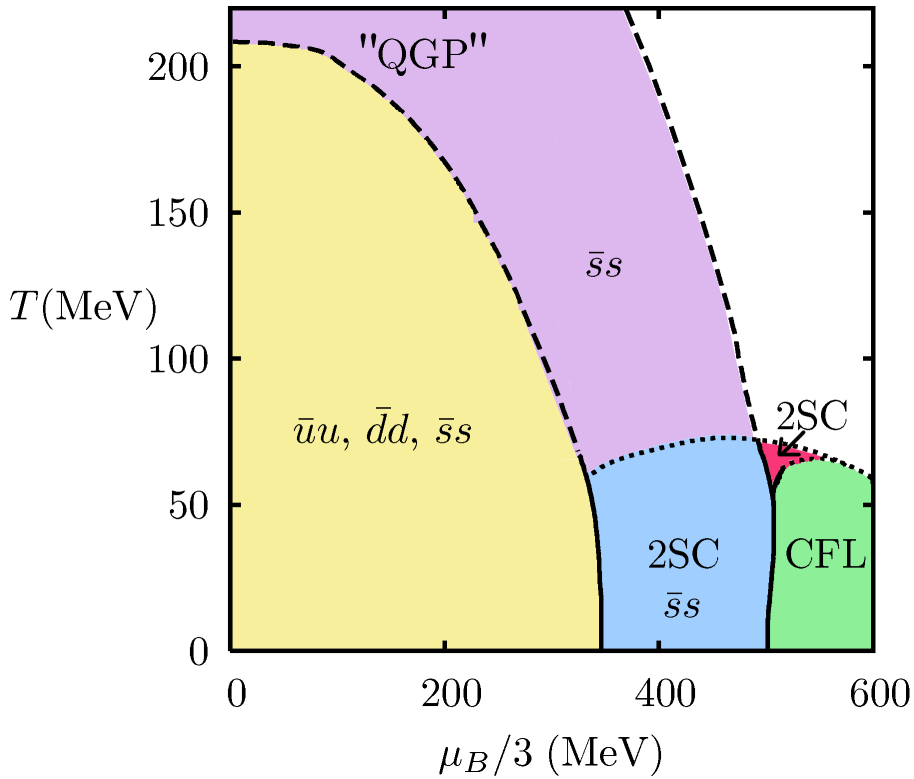
Color superconductivity vs. pseudoscalar condensation in a three-flavor NJL model

HW, D. Boer and J.O. Andersen, Phys. Rev. **D 72** 014015 (2005)



$m_{0u}, m_{0d} = 5$ MeV
$m_{0s} = 112$ MeV

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Outline

- Introduction
- The NJL model
- Effective potential
- Condensates
- Phase diagrams
- Conclusions

Color superconductivity vs. pseudoscalar condensation

Color-superconductivity: large μ's and $\mu_u \approx \mu_d$, $\mu_u \approx \mu_s$, $\mu_d \approx \mu_s$

Study of phase diagrams with different μ's:

CSC with different μ's	Gastineau et al. '02, Neumann et. al. '03 ...
Neutralization (color μ)	Alford and Rajagopal '02, ...
Recent results compact stars	Rüster et al. '05, Blaschke et. al. '05

Pseudoscalar condensation: $\langle \bar{u} i \gamma_5 d \rangle \neq 0$, large μ's and $\mu_u \approx -\mu_d$, $\mu_u \approx -\mu_s$

Study of phase diagrams with different μ's:

Pseudoscalar cond. in QCD	Son and Stephanov '01
$N_f = 2$ with different μ's	Barducci et al. '04, Toublan and Kogut '04
$N_f = 3$ with different μ's	Barducci et al. '05

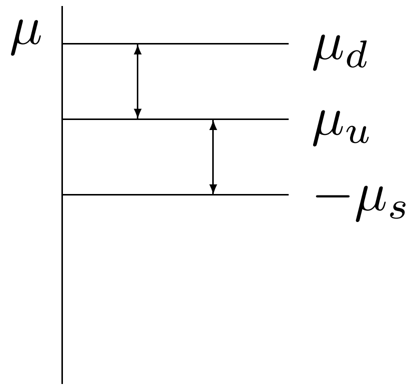
Color superconductivity vs. pseudoscalar condensation

Color-superconductivity: large μ's and $\mu_u \approx \mu_d$, $\mu_u \approx \mu_s$, $\mu_d \approx \mu_s$

Pseudoscalar condensation: $\langle \bar{u} i \gamma_5 d \rangle \neq 0$, large μ's and $\mu_u \approx -\mu_d$, $\mu_u \approx -\mu_s$

Questions:

- Phase with CSC and pseudoscalar condensation?
  for example if $\mu_u \approx \mu_d$ and $\mu_u \approx -\mu_s$
- Calculate phase diagrams in μ_u, μ_d, μ_s and T plane.



A 3-flavor Nambu-Jona–Lasinio (NJL) model

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - M_0 + \mu\gamma_0\right)\psi + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$$

$$M_0 = \begin{pmatrix} m_{0u} & & \\ & m_{0d} & \\ & & m_{0s} \end{pmatrix} \quad \mu = \begin{pmatrix} \mu_u & & \\ & \mu_d & \\ & & \mu_s \end{pmatrix}$$

taking only attractive scalar and pseudoscalar channels

$$\mathcal{L}_{\bar{q}q} = G\left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2\right] \qquad a = 0\ldots 8.$$

$$\mathcal{L}_{qq} = \frac{3}{4}G(\psi^T t_A\lambda_B Ci\gamma_5\psi)(\bar{\psi}t_A\lambda_B Ci\gamma_5\bar{\psi}^T) \qquad A, B \in \{2,5,7\}$$

Antisymmetric channel is attractive, symmetric repulsive.

$$G = 2.3/\Lambda^2 \quad \Lambda = 600\ \mathrm{MeV} \quad m_{0u} = m_{0d} = 5.5\ \mathrm{MeV} \quad m_{0s} = 112\ \mathrm{MeV}$$

Good prediction meson masses (Klevanksy '92, ...)

Symmetries of the NJL model

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - M_0 + \mu\gamma_0\right)\psi + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$$

NJL, low-energy effective model for QCD

Symmetry structure similar to QCD but global $\mathrm{SU}(3)_c$ symmetry

- $m_{0u} = m_{0d} = m_{0s} = 0$	$\mu_u = \mu_d = \mu_s$	$\mathrm{U}(3)_L \times \mathrm{U}(3)_R$
		$\downarrow$
- $m_{0u} = m_{0d} = m_{0s}$	$\mu_u = \mu_d = \mu_s$	$\mathrm{U}(3)_V$
		$\downarrow$
- $m_{0u} \neq m_{0d} \neq m_{0s}$	$\mu_u \neq \mu_d \neq \mu_s$	$\mathrm{U}(1)_B \times \mathrm{U}(1)_I \times \mathrm{U}(1)_Y$

The effective potential

- Introduce auxiliary fields
- Shift auxiliary fields, action becomes quadratic in fermion fields
- Integrate over fermion fields $\bar{\psi}, \psi$
- Replace auxiliary fields by space-time independent vev's
  (mean field approximation)

Result: vev's auxiliary fields related to condensates

$\alpha_a = -2G \left\langle \bar{\psi} \lambda_a \psi \right\rangle$	chiral condensates, $\bar{u}u$, $\bar{d}d$, $\bar{s}s$
$\beta_a = -2G \left\langle \bar{\psi} \lambda_a i \gamma_5 \psi \right\rangle$	pseudoscalar condensation
$\Delta_{AB} \leftrightarrow \frac{3}{2} G \left\langle \psi^T t_A \lambda_B C \gamma_5 \psi \right\rangle$	scalar diquark condensation
	color superconductivity

The effective potential

chiral → pseudoscalar → diquark ← condensates

$$\mathcal{V} = \frac{{\alpha_a}^2 + {\beta_a}^2}{4G} + \frac{|\Delta_{AB}|^2}{3G} - \frac{T}{2} \sum_{p_0=(2n+1)\pi T} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \log \det K$$

$$K = \begin{pmatrix} \mathbb{1}_c \otimes \mathcal{D}_1 & \Delta_{AB}\, t_A \otimes \lambda_B \otimes \gamma_5 \\ -\Delta^*_{AB}\, t_A \otimes \lambda_B \otimes \gamma_5 & \mathbb{1}_c \otimes \mathcal{D}_2 \end{pmatrix}$$

$$\mathcal{D}_1 = \mathbb{1}_f \otimes (i\gamma_0 p_0 + \gamma_i p_i) - \mu \otimes \gamma_0 - (M_0 + \alpha_a \lambda_a) \otimes \mathbb{1}_d - \beta_a \lambda_a \otimes i\gamma_5$$

$$\mathcal{D}_2 = \mathbb{1}_f \otimes (i\gamma_0 p_0 + \gamma_i p_i) + \mu \otimes \gamma_0 - (M_0 + \alpha_a \lambda_a^T) \otimes \mathbb{1}_d - \beta_a \lambda_a^T \otimes i\gamma_5$$

$K = 4N_c N_f \times 4N_c N_f = 72 \times 72$ dimensional matrix

Some technicalities

$$\mathcal{V} = \frac{{\alpha_a}^2 + {\beta_a}^2}{4G} + \frac{|\Delta_{AB}|^2}{3G} - \frac{T}{2} \sum_{p_0=(2n+1)\pi T} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \log \det K$$

- Simplify determinant K

$$\det \begin{pmatrix} a & 0 & b \\ 0 & 0 & c \\ 0 & d & 0 \end{pmatrix} = \det \begin{pmatrix} a & b & 0 \\ 0 & c & 0 \\ 0 & 0 & d \end{pmatrix} = \det \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \times d$$

- Summing over Matsubara frequencies $(2n+1)\pi T$ can be done exactly
  by determination of eigenvalues matrix related to K
  eigenvalues analytic in special cases, for general case: LAPACK

- Integration over three momentum p up to cutoff Λ

- Minimization with respect to α_a, β_a and Δ_{AB} using MINUIT

Chiral condensates

Chiral condensates are: $\alpha_a = -2G \left\langle \bar{\psi} \lambda_a \psi \right\rangle$

It turns out that only α_0, α_3 and α_8 can be nonzero.

Linear combination of α's gives $\langle \bar{u}u \rangle$, $\left\langle \bar{d}d \right\rangle$ and $\langle \bar{s}s \rangle$.

Condensates like $\langle \bar{u}d \rangle$ do not arise (checked numerically).

$\alpha_{0,3,8}$ favored over $\beta_{0,3,8} \longrightarrow$ QCD inequalities (Weingarten, '82)
$\alpha_{1,2,4,5,6,7}$ disfavored over $\beta_{1,2,4,5,6,7} \longrightarrow$ instantons

Typical values ($T = 0$, $\mu = 0$)

$$\langle \bar{u}u \rangle = \left\langle \bar{d}d \right\rangle = -(304 \text{ MeV})^3 \qquad \langle \bar{s}s \rangle = -(324 \text{ MeV})^3$$

Pseudoscalar condensation

Pseudoscalar condensates are: $\beta_a = -2G \left\langle \bar{\psi} \lambda_a i \gamma_5 \psi \right\rangle$

It turns out that only β_1, β_2, β_4, β_5, β_6 and β_7 can be nonzero

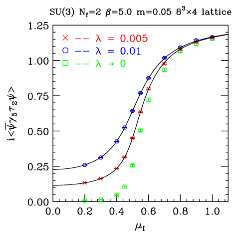
Use of $\mathrm{U}(1)$ flavor symmetries gives nonzero values for

β_2	$\langle \bar{u} i \gamma_5 d \rangle$	pion $(\pi^{\pm})$ condensation
β_5	$\langle \bar{u} i \gamma_5 s \rangle$	charged kaon $(K^{\pm})$ condensation
β_7	$\left\langle \bar{d} i \gamma_5 s \right\rangle$	neutral kaon $(K^0/\bar{K}^0)$ condensation

These condensates break parity!

Pion condensation can arise if $|\mu_u - \mu_d| > m_\pi$ — Son and Stephanov '01
Kaon condensation can arise if $|\mu_{u,d} - \mu_s| > m_K$ — Toublan and Kogut '01

$\langle \bar{u} i \gamma_5 u \rangle$, $\left\langle \bar{d} i \gamma_5 d \right\rangle$ and $\langle \bar{s} i \gamma_5 s \rangle$ condensates do not arise (checked numerically)

Also found in $N_f = 2$ lattice QCD 

Kogut and Sinclair '02

Color-superconductivity

Diquark condensates are: $\Delta_{AB} \leftrightarrow \langle \psi^T t_A \lambda_B C\gamma_5 \psi \rangle$

We did not neutralize CSC phases with color chemical potentials
Blaschke et al. '05, 2SC with $\Delta_{22} = \Delta_{52} = \Delta_{72}$ is also neutral and favored.

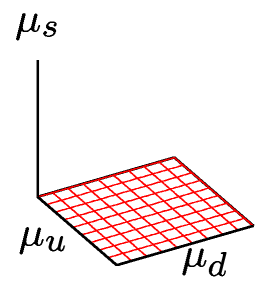
$\mathrm{SU}(3)$ color symmetries $\rightarrow$ rotate condensates away
$\mathrm{SU}(3)$ color and flavor $\mathrm{U}(1)$ symmetries $\rightarrow$ condensates become real

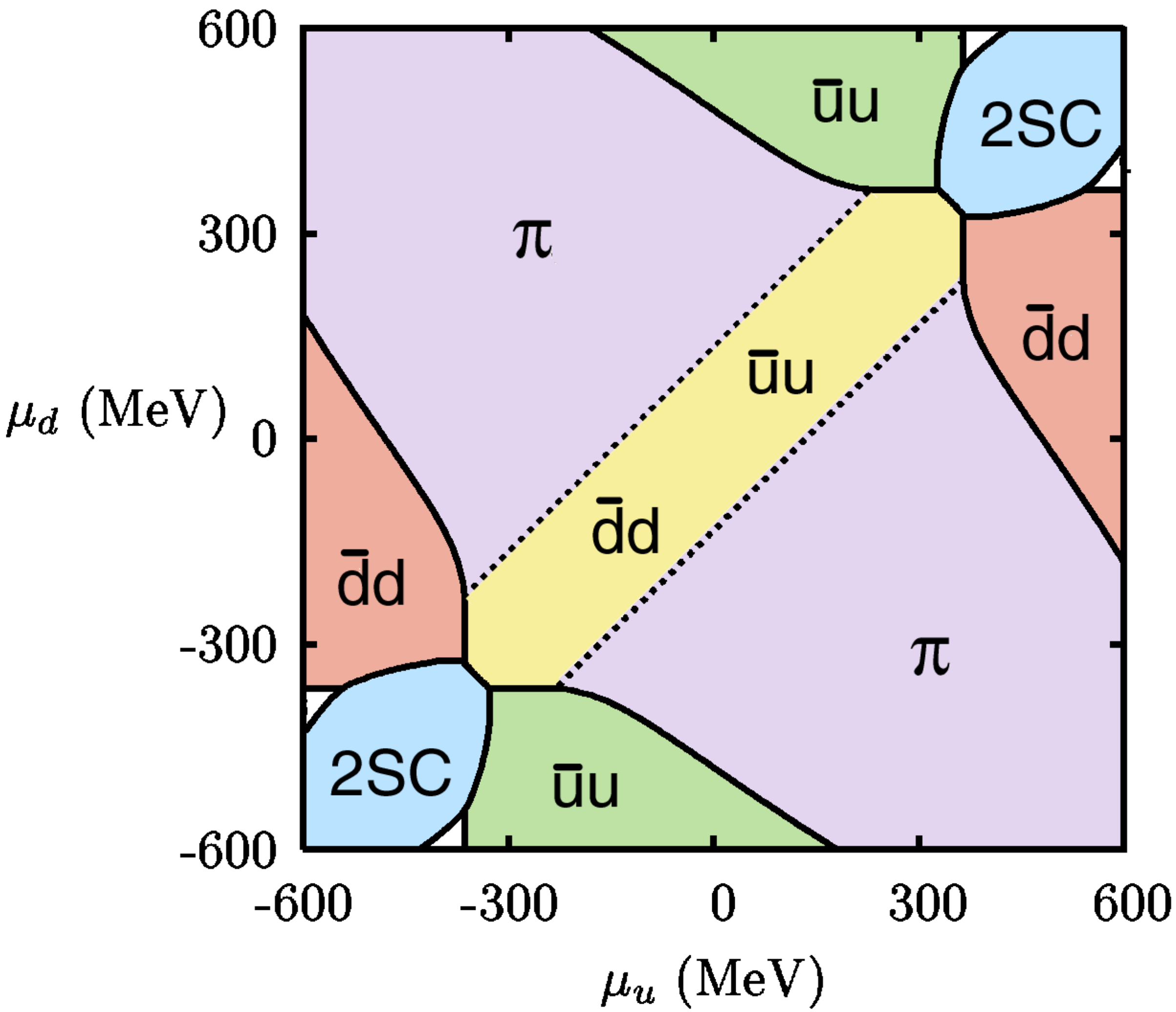
Condensation at low T and large μ's if $\mu_u \approx \mu_d$, $\mu_u \approx \mu_s$, $\mu_d \approx \mu_s$

$\Delta_{22} \neq 0$, $\Delta_{55} \neq 0$, $\Delta_{77} \neq 0$	CFL	Alford et al. '99
$\Delta_{77} = 0$, $\Delta_{22} \neq 0$, $\Delta_{55} \neq 0$	uSC	
$\Delta_{55} = 0$, $\Delta_{22} \neq 0$, $\Delta_{77} \neq 0$	dSC	
$\Delta_{22} = 0$, $\Delta_{55} \neq 0$, $\Delta_{77} \neq 0$	sSC	s-quark always pairs
$\Delta_{22} \neq 0$, $\Delta_{55} = 0$, $\Delta_{77} = 0$	2SC	Bailin and Love '84
$\Delta_{55} \neq 0$, $\Delta_{22} = 0$, $\Delta_{77} = 0$	2SCus	
$\Delta_{77} \neq 0$, $\Delta_{22} = 0$, $\Delta_{55} = 0$	2SCds	

Other diquark condensates do not arise (checked numerically)

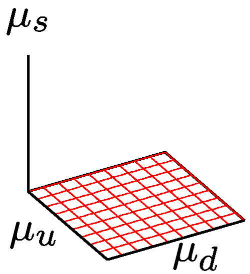
Phase diagram $T = 0$, $\mu_s = 0$, μ_u vs. μ_d



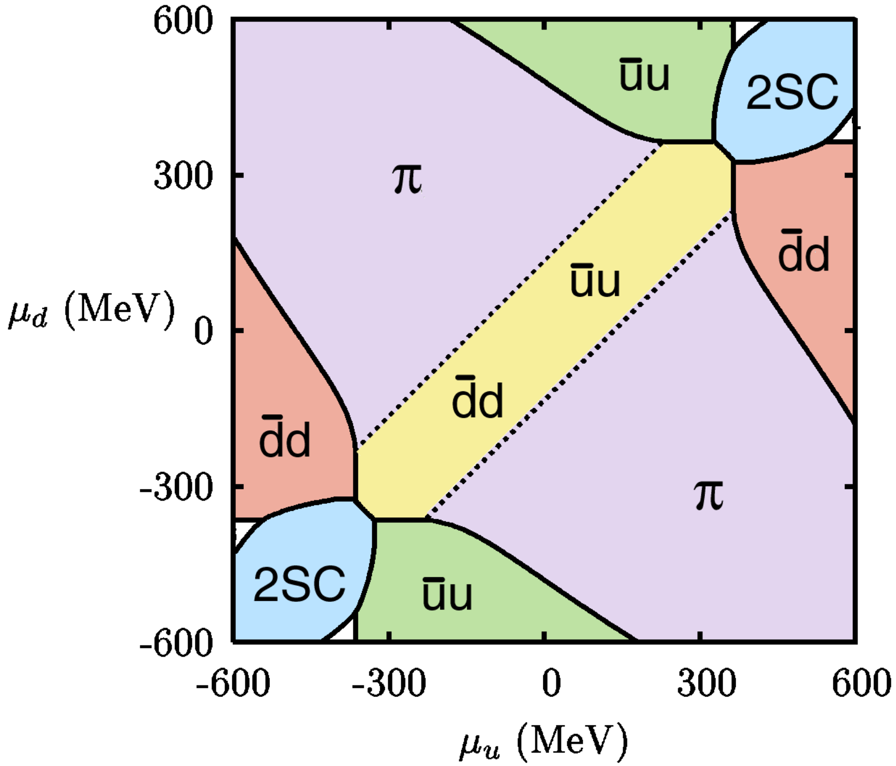


- $\mu_s = 0$, $\bar{s}s$ everywhere
- $q\ \bar{q}$ symmetry
  $\mu_u, \mu_d \to -\mu_u, -\mu_d$
- transition $\bar{u}u + \bar{d}d \to \pi$
  diagonal line, 2nd order
  at $|\mu_u - \mu_d| = m_\pi$
- 1st order to 2SC
- π and 2SC separated
  by 2 transitions
- If $|\mu_u - \mu_d| > 30$ MeV,
  two 2nd order transitions
  before entering 2SC

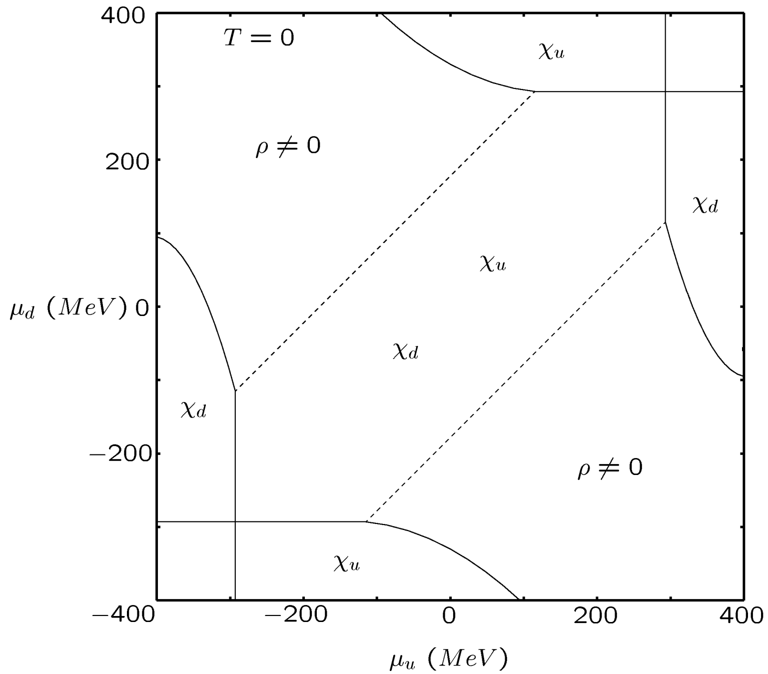
Phase diagram $T = 0$, $\mu_s = 0$, μ_u vs. μ_d



Calculation with 2SC

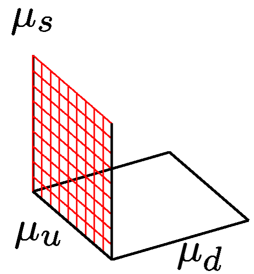


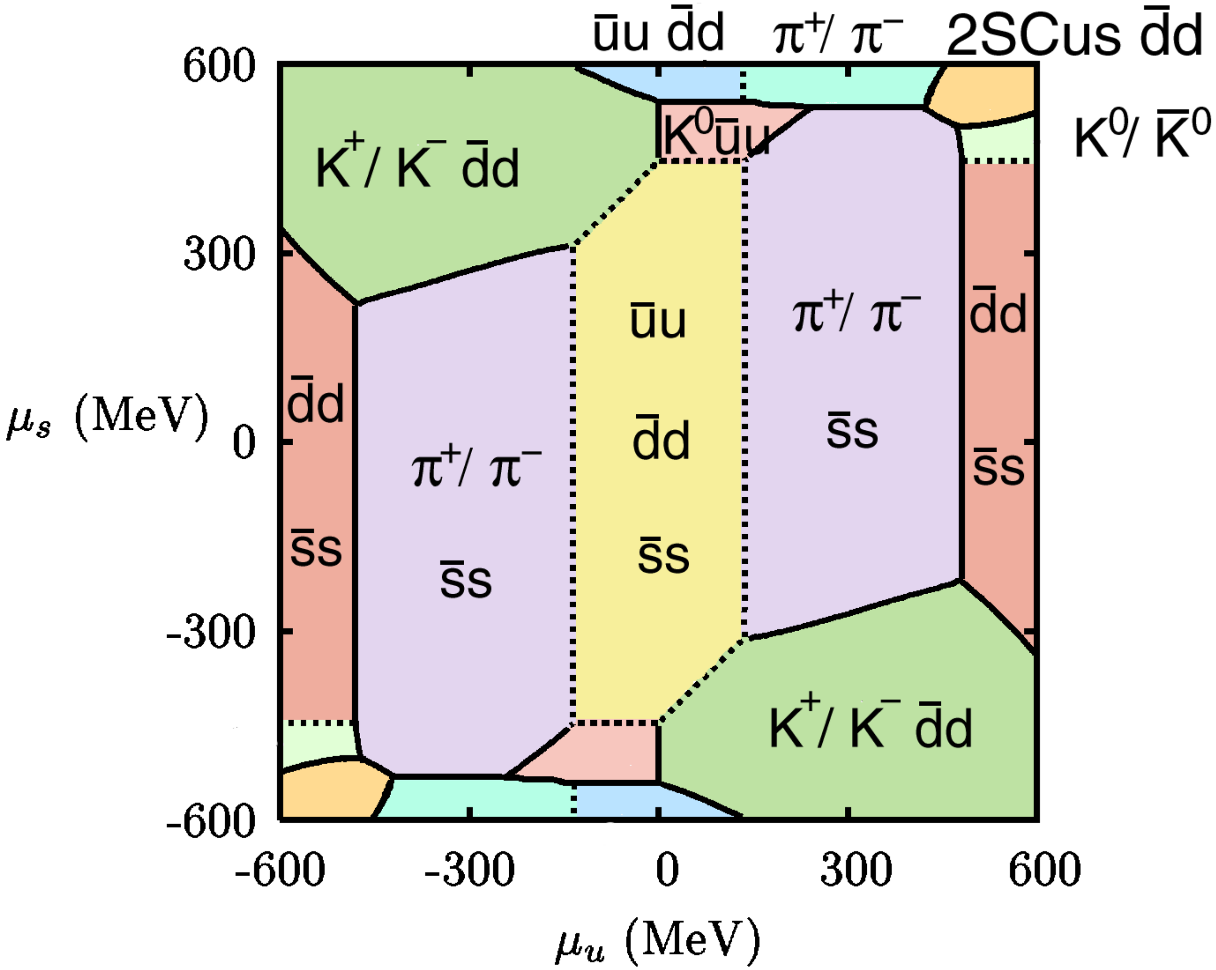
Calculation without 2SC



Barducci et al. '04

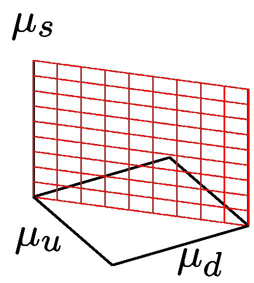
Phase diagram $T = 0$, $\mu_d = 0$, μ_u vs. μ_s

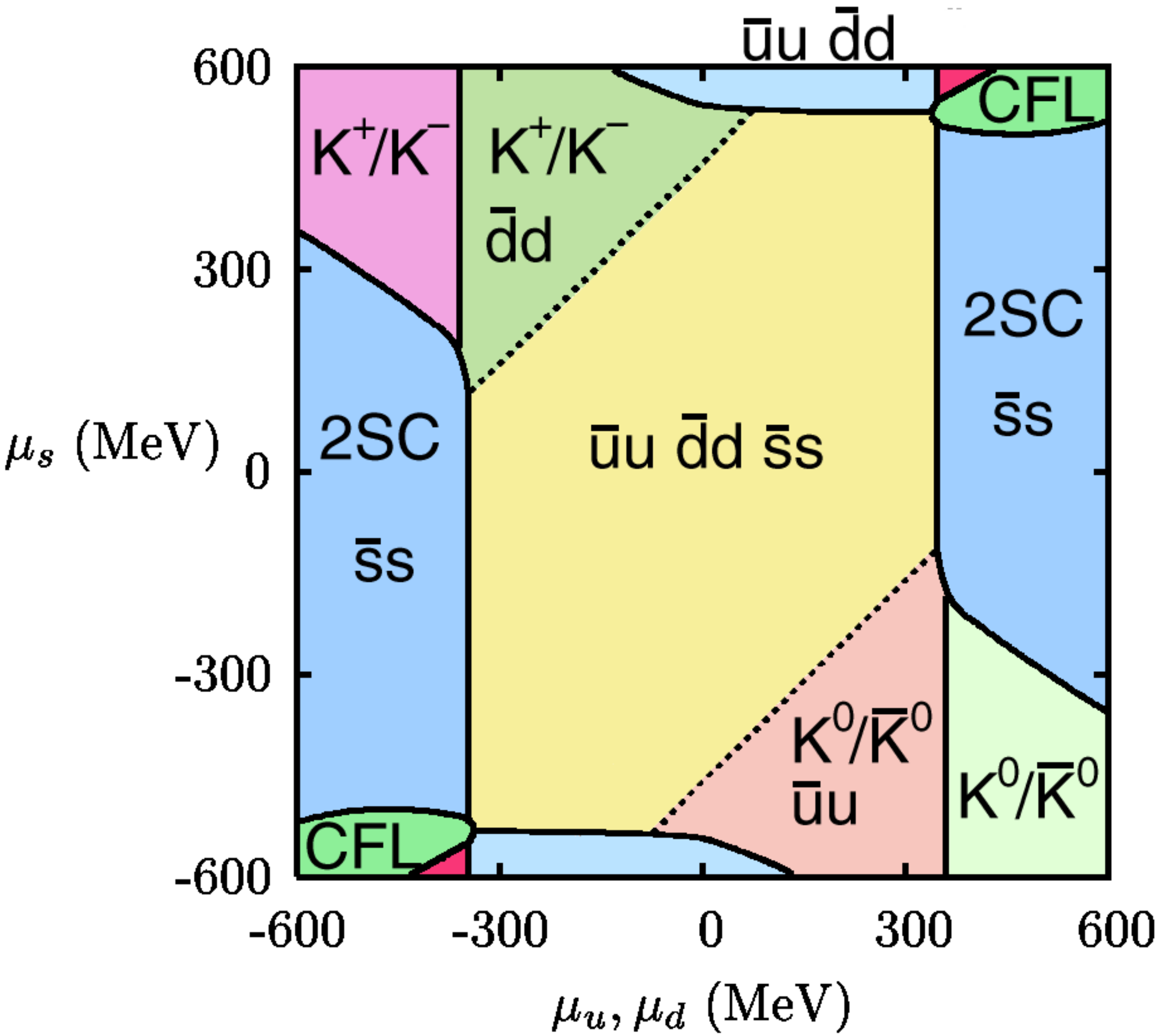




- $q \leftrightarrow \bar{q}$ symmetry
  $\mu_u, \mu_s \to -\mu_u, -\mu_s$
- competition between pseudoscalar condensation and CSC
- 2SCus and $K^0, \bar{K}^0$
  1st order transition
- 2SCus and π
  1st order transition
- π w/o $\bar{s}s$
- π if $|\mu_u - \mu_d| > m_\pi$
- K if $|\mu_{u,d} - \mu_s| > m_K$

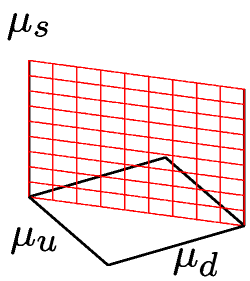
Phase diagram $T = 0$, $\mu_u = \mu_d + \epsilon$ vs. μ_s



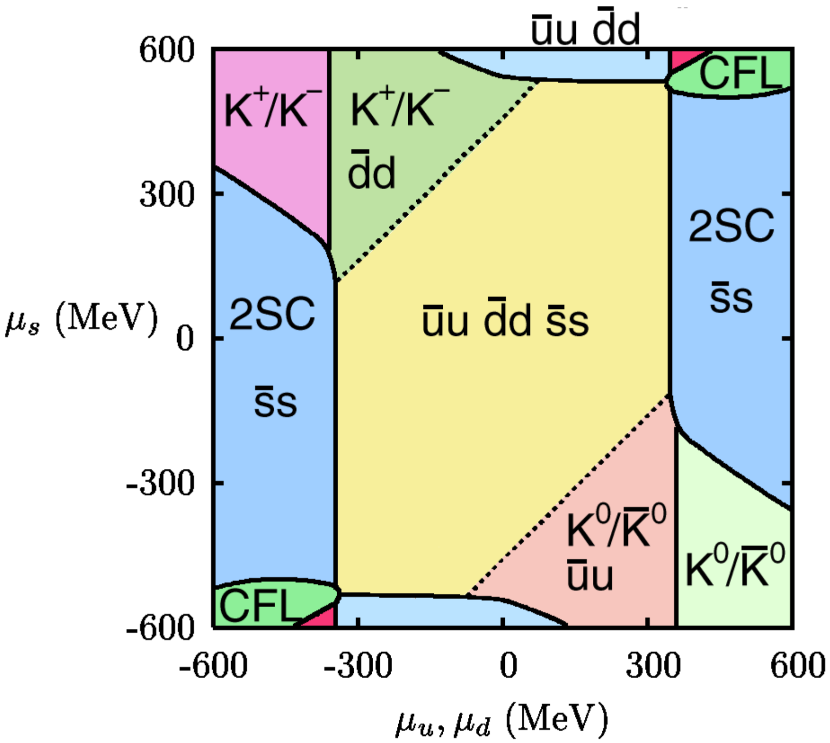


- $q \leftrightarrow \bar{q}$ symmetry
- 2SC and K separated by 1st order transition
- diagonal line, 2nd order at $|\mu_{u,d} - \mu_s| = m_K$
- 1st order to 2SC
- 1st order to CFL
- possible to go via 1 trans. from vac. to CFL
- K w/o $\bar{u}u$ / $\bar{d}d$

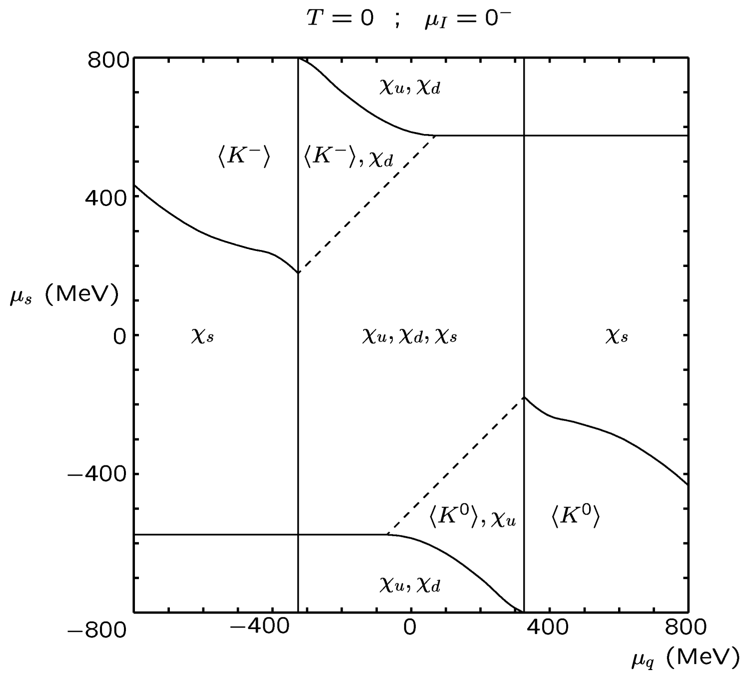
Phase diagram $T = 0$, $\mu_u = \mu_d + \epsilon$ vs. μ_s



Calculation with CSC

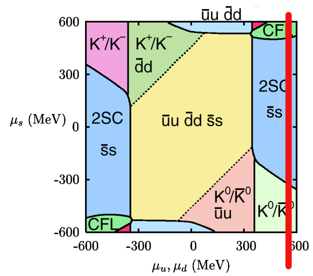


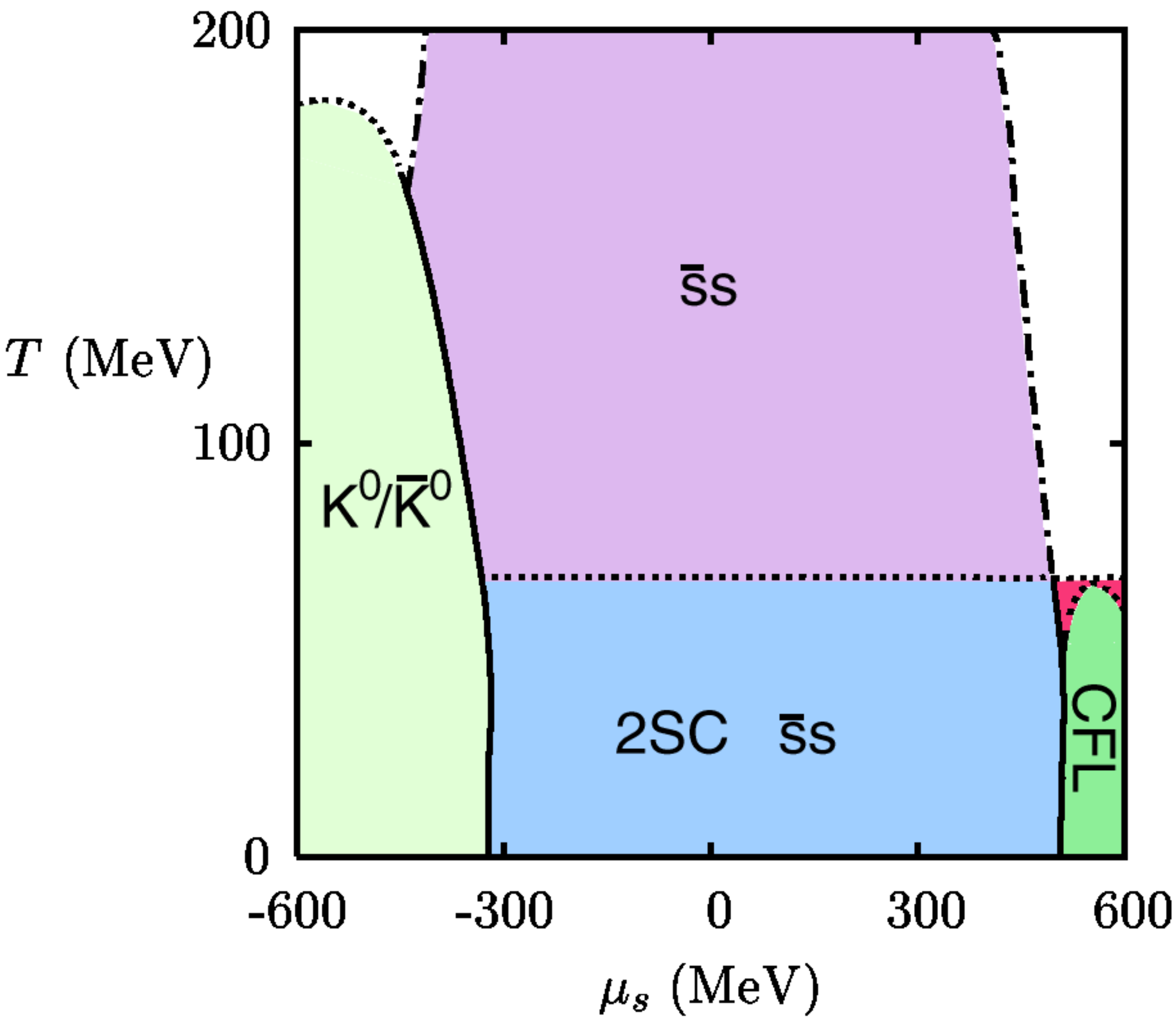
Calculation without CSC



Barducci et al. '05

Phase diagram $\mu_u = \mu_d = 550$ MeV, μ_s vs. T





- $K^0/\bar{K}^0$ separated from 2SC by 1st order transition for all T
- K^0 to restored phase 2nd order transition high T
- cross-over out $\bar{s}s$
- 2nd order out 2SC
- K^0 and 2SC separated by 1st order transition at T
- 3 critical endpoints

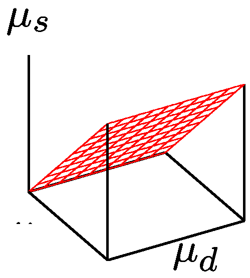
Conclusions

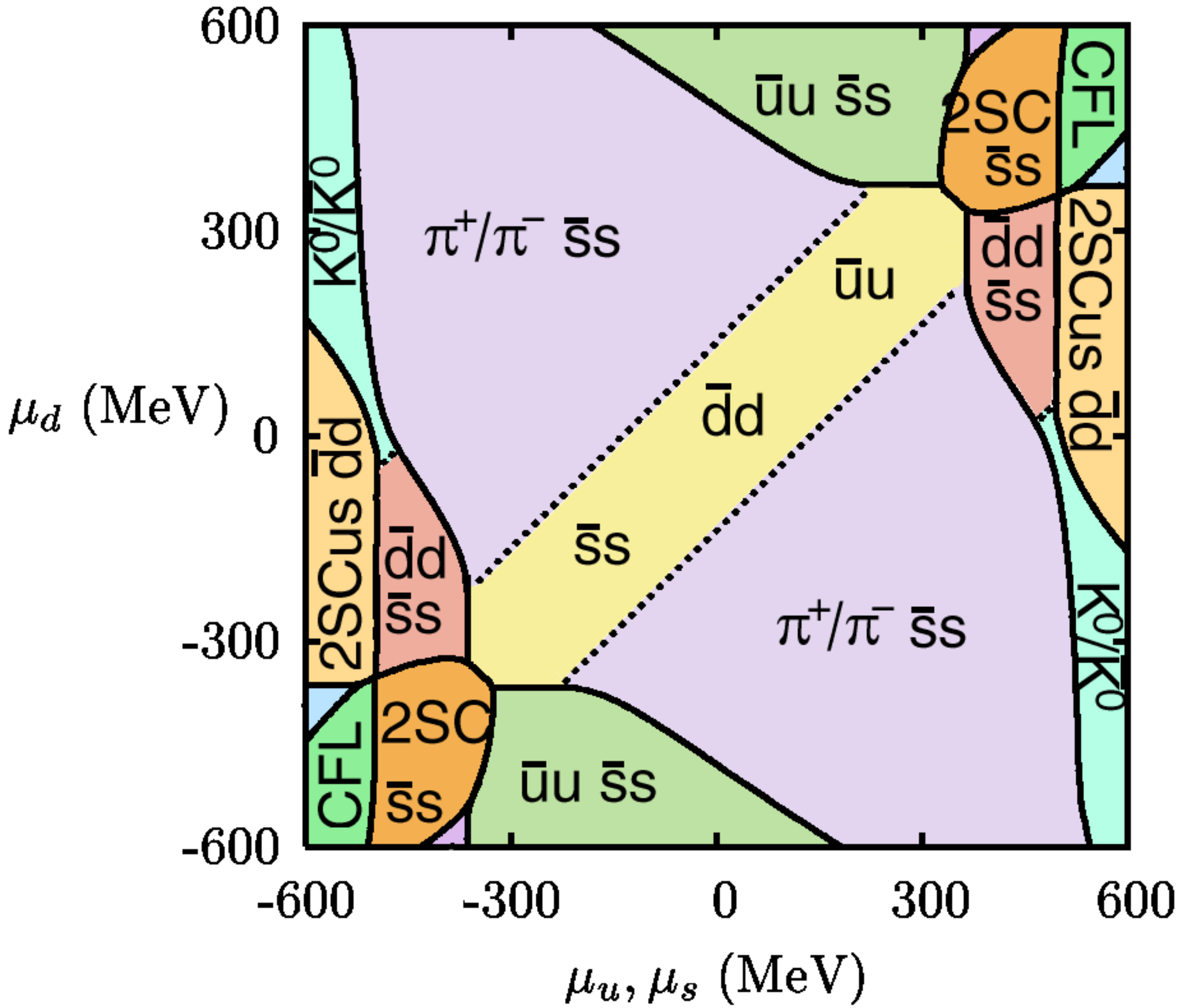
- Phase diagrams NJL model for different chemical potentials with color-superconductivity and pseudoscalar condensation
- Pseudoscalar condensed separated from color-superconducting phase by a first order transition

Outlook

- Include instanton induced interaction
- Include pseudoscalar diquark condensation
- Gapless phases
- LOFF phase
- Neutralization w/o color chemical potentials

Phase diagram $T = 0$, $\mu_u = \mu_s$ vs. μ_d





Phase diagram $\mu_u = \mu_s = 550$ MeV, μ_d vs. T