Trial for simulation of real time non-equilibrium dynamics

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Features

Basis: Construct the (quantum) ensemble by a stochastic process

in euclidean formulation: well defined: stochastic process in a 5-th ("Langevin") time

- Parisi Wu stochastic quantization using Langevin equation :

- proofs of equivalence with path integral formulation, proofs of convergence, step-size dependence, etc [see also Damgaard and Hueffel eds., Stochastic Quantization]

- simulation of non-compact Y-M theory with stochastic gauge fixing [E.Seiler, I.O.S., D.Zwanziger] (generally for Y-M simulations: convergence is achieved in the orbit space, hence for gauge invariant observables, while the fields themselves drift irreversibly)

in real time (Minkowski) formulation:

- reformulation of the stochastic quantization for the Minkowski path integral: Langevin equation with complex driving force

- implementation of the Nelson quantization procedure *directly in the real (physical) time*: no 5-th dimension, but needs the ground state wave function to define the driving force

- in both cases (more or less formal) proofs of convergence and equivalence to the path integral formulation under certain conditions [Hueffel and Rumpf, Okamoto, etc] (the convergence is shown in the sense of tempered distributions, e.g. for correlation functions)

Our approach: Langevin equation with complex driving force for N-E dynamics

- Problem definition: fix the initial (in real time) configuration

- at all *later times* use the Langevin equation to simulate quantum fluctuations

- use ergodicity of the stochastic processes to take the expectation values of correlation functions over the *Langevin trajectories*

Studies (many prompted by the Parisi Wu quantization), e.g.:

G. Parisi, P.L. 131B (83) 383: arguments for using complex Langevin simualtions

J. Klauder and W. Petersen, J. Stat.Ph. 39 (85) : convergence of the complex Langevin equation

P. Damgaard and H. Hueffel eds "Stochastic Quantization" 1986 (collection of articles)

H. Hueffel and H. Rumpf, P.L. 148B (84) 104, complex Langevin for ϕ^4

E. Seiler, Schladming 1984, comprehensive study of the Nelson and stochatic quantization, Y-M theory, stochastic gauge fixing

R. Streater, Schladming 1984, stochastic integrals

E. Seiler, I.O.S. and D. Zwanziger N.Ph. B239 (84) 177, Y-M simulations with stochastic gauge fixing

Newer studies (Okamoto, Schueltke, Okano, ...)

Model: ϕ^4 theory in 4-D to check the ideas

Physical S-T: $x = \{\vec{x}, t\}$; Langevin "time" ω; Fields: $\phi(\vec{x}, t; \omega)$

$$-\frac{i}{h} W[\phi] = -\frac{i}{h} \int d^4x \left(\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 \right) \qquad (1)$$

$$(2)$$

leads to the Langevin equation

$$\begin{aligned}
\partial_\omega \phi &= -\frac{i}{h} \frac{\delta}{\delta\phi} W[\phi] \\
&= -\frac{i}{h} \left((\partial_0^2 - \partial_{\vec{x}}^2)\phi + m^2\phi + \lambda\phi^3 \right) + \eta \qquad (3)
\end{aligned}$$

with gaussian noise term

$$< \eta(x;\omega)\eta(x';\omega') > = 2(2\pi)^4 \delta^4(x - x')\delta(\omega - \omega') \qquad (4)$$

[Hueffel and Rumpf]

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On the lattice: $N_s^3 . N_t$

Anisotropic space-time discretization a, a_t: $\quad a/a_t = \gamma$

Langevin discretization: $d\omega$

$$\Phi = a\phi; \quad M = am; \quad \tilde{\omega} = a^{-2}\omega; \quad \epsilon = a^{-2}d\omega \qquad (5)$$

$$\tilde{\eta} = \left(a^{\frac{3}{2}} a_t^{\frac{1}{2}} d\omega \right) \eta; \quad < \tilde{\eta}(x, \tilde{\omega}), \tilde{\eta}(x', \tilde{\omega}') > = 2\delta_{x,x'}\delta_{\tilde{\omega},\tilde{\omega}'} \qquad (6)$$

$$\Phi(\tilde{\omega} + \epsilon) = \Phi(\tilde{\omega}) - \frac{i}{h}\epsilon \sum_x \left((\gamma^2 \Delta_0^2 - \sum_i \Delta_i^2) - M^2\Phi - \lambda\Phi^3 \right)$$

$$+ \sqrt{\epsilon\gamma}\, \tilde{\omega} \qquad (7)$$

Notice: distributions depend on the discretization step ϵ .

Lattices: $8^3.20$, $\gamma = 4$

Boundary conditions:

- fixed initial configuration $\Phi(\vec{x}, t = 0)$ for all $\tilde{\omega}$

- free b.c. at $t = N_t$

- p.b.c. in spatial directions.

Starting configuration for the Langevin process:

- classical solution corresponding to the initial configuration,
$\Phi(\vec{x}, t > 0; \tilde{\omega} = 0) = \Phi_{class}(\vec{x}, t > 0)$

- other configurations, e.g., $\Phi(\vec{x}, t > 0; \tilde{\omega} = 0) = 0$.

Calculate the correlations

$$C(t) \;=\; < \frac{1}{N_s^3} \sum_{\vec{x}} \Phi(\vec{x},0)\Phi(\vec{x},t)$$

$$-\frac{1}{N_s^3} \sum_{\vec{x}} \Phi(\vec{x},0)\frac{1}{N_s^3} \sum_{\vec{x}}{}' \Phi(\vec{x}',t) > \tag{8}$$

$$G(t) \;=\; < \frac{1}{N_s^3} \sum_{\vec{x}} \Phi(\vec{x},0)\Phi(\vec{x},t) > \tag{9}$$

Free field

$\lambda = 0$, various M

8^3.20, init/start/fin=10f, mink , mass=2, gamma=4, it=   0
it=   20
it=   50
it= 100
it= 300
it= 1000
it= 2000
it= 3000
it= 4000
it= 5000
0.
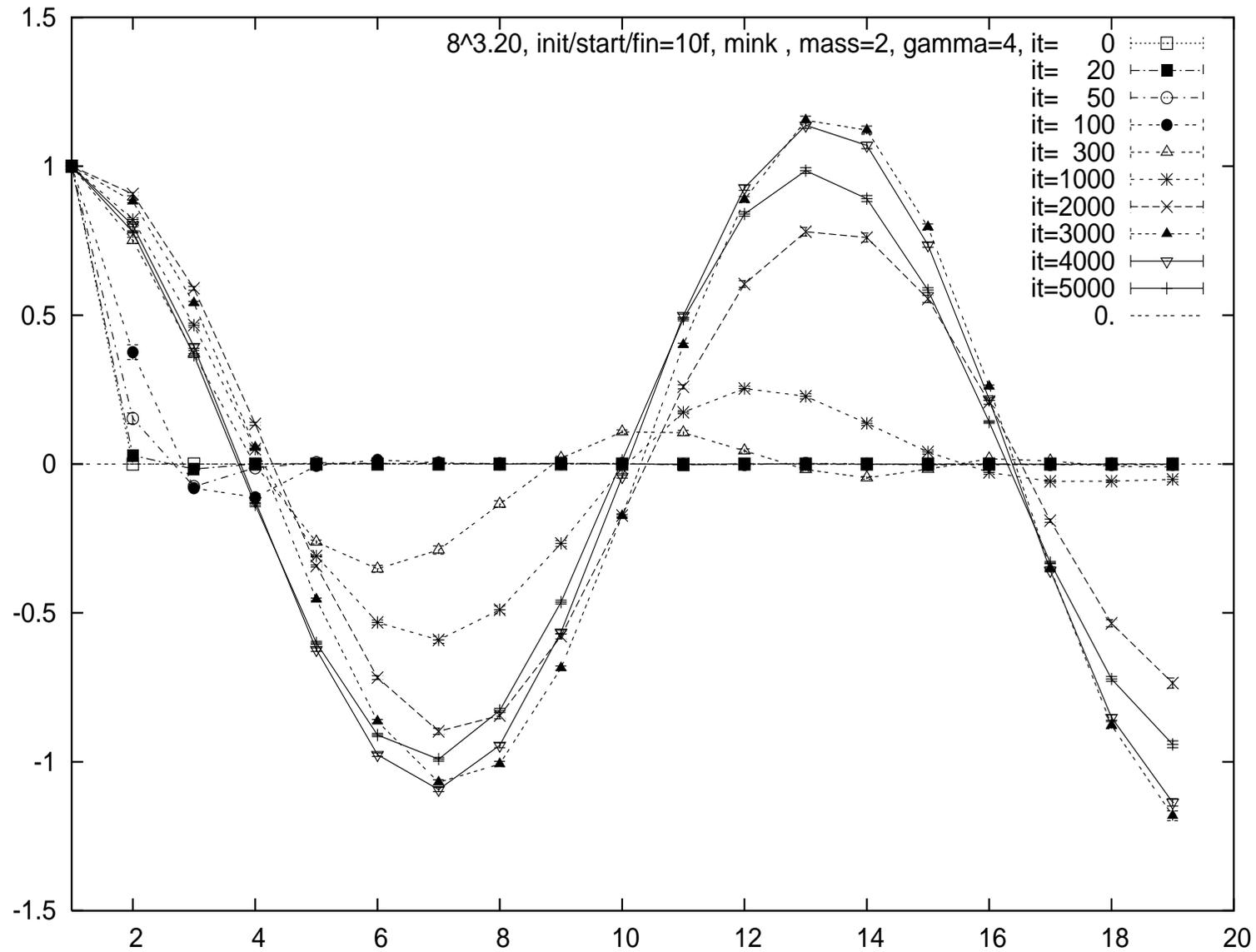
Figure 1: $ReG(t), M = 2, \lambda = 0.$
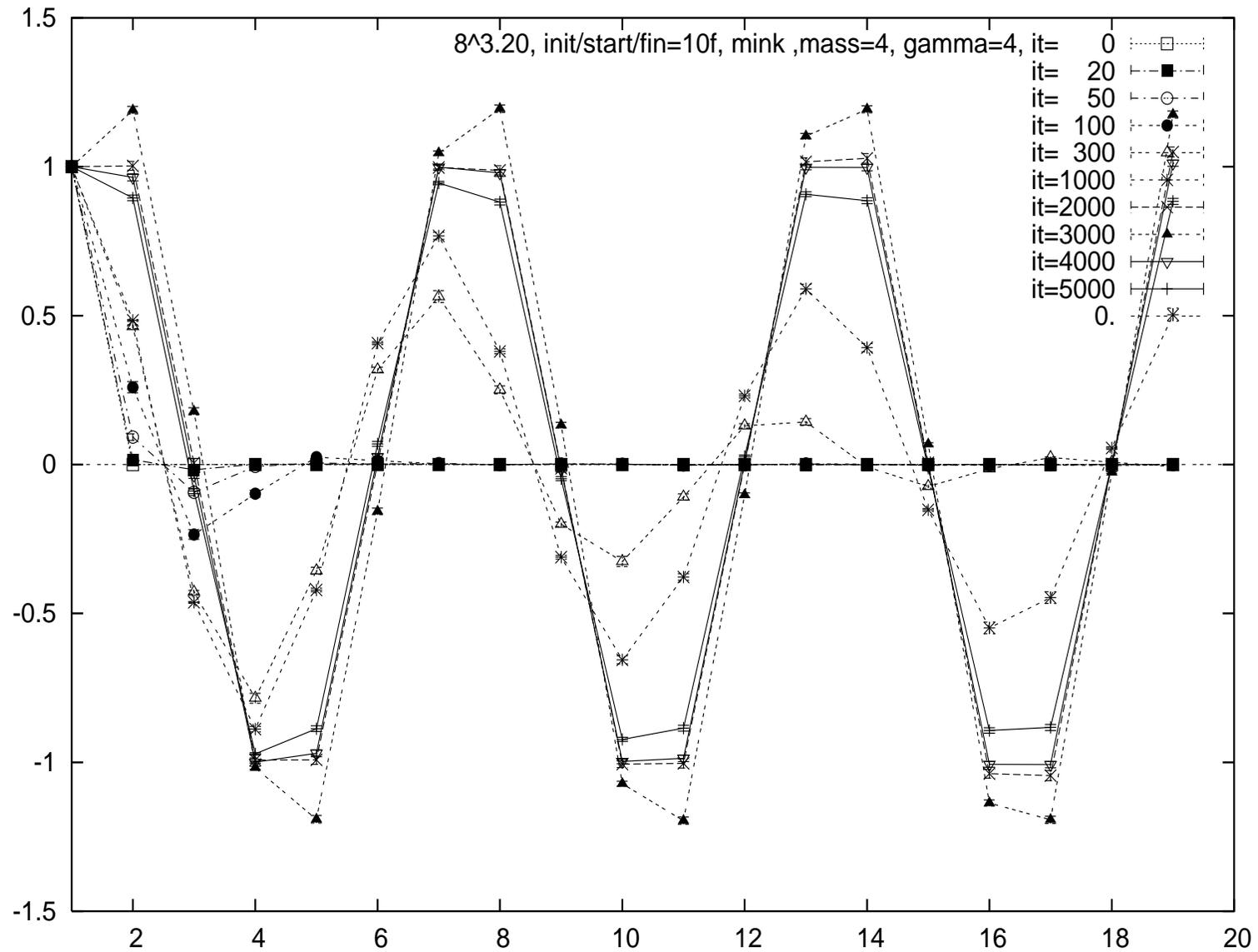
Figure 2: $ReG(t), M = 4, \lambda = 0.$

Massless ϕ^4
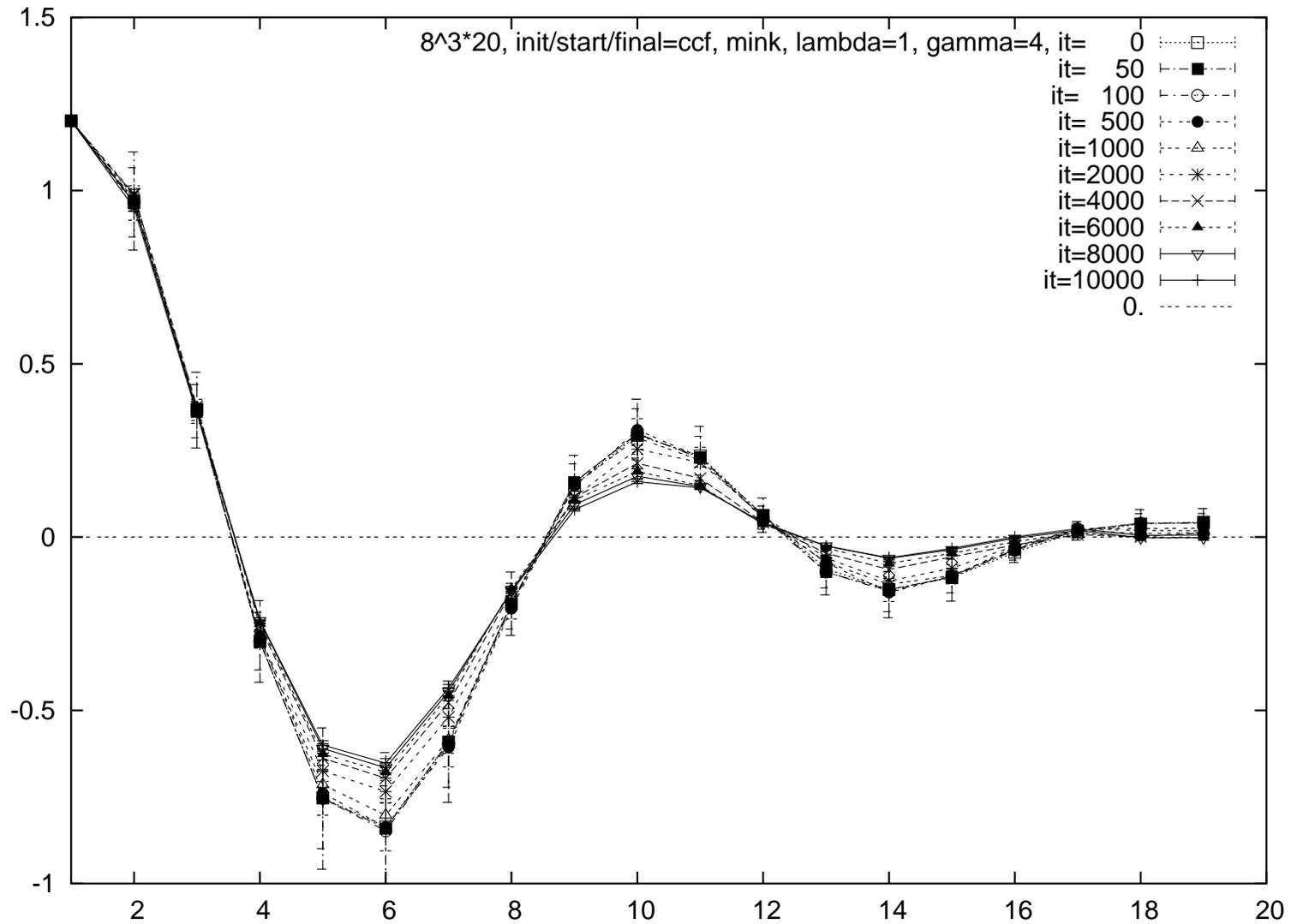
Figure 3: $ReC(t), M = 0, \lambda = 1$, classical start.

Figure 4: $ReC(t), M = 0, \lambda = 1$, 0 start.
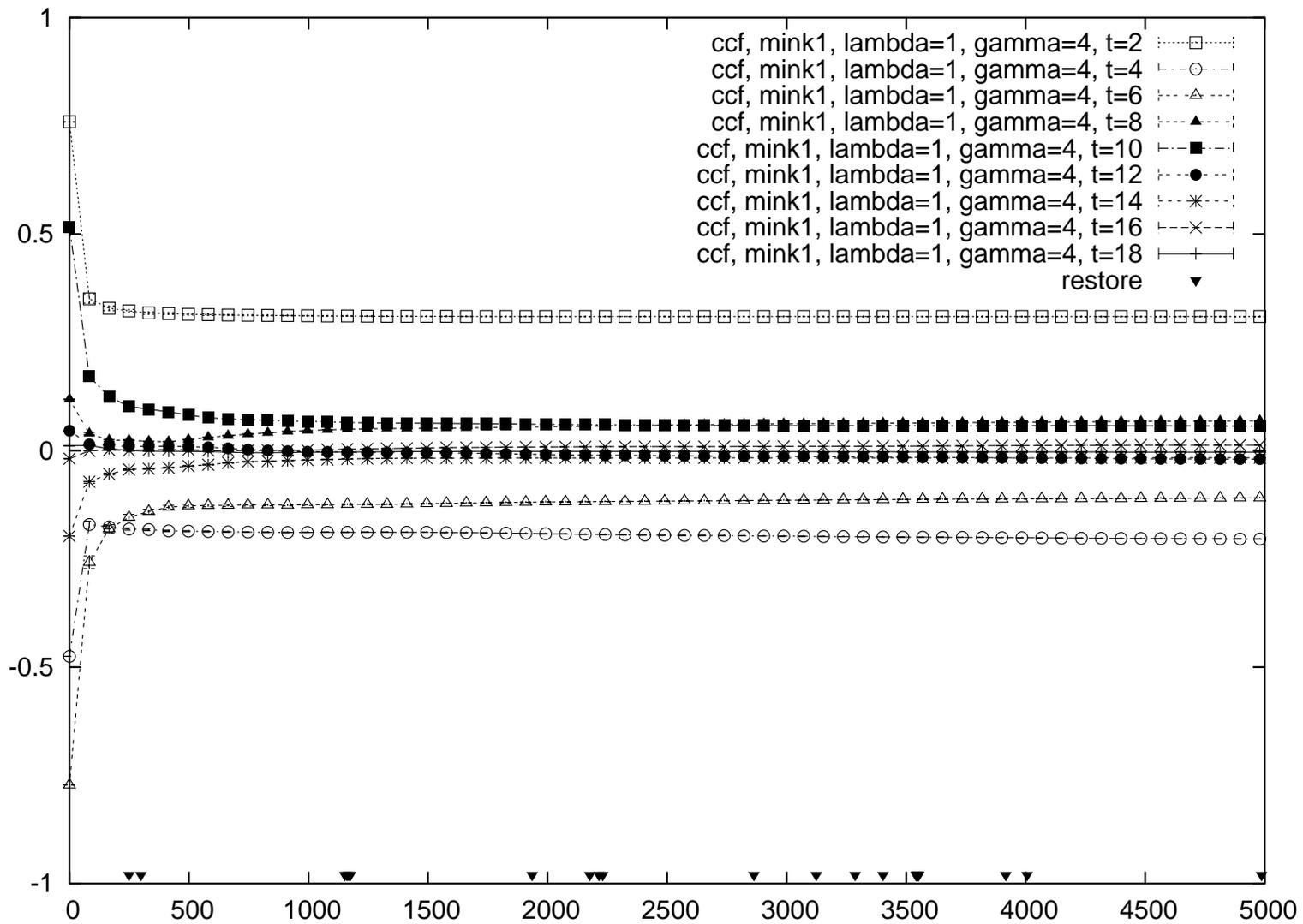
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Figure 5: $ReC(t), M = 0, \lambda = 1$.

The legend of the figure reads:

ccf, mink1, lambda=1, gamma=4, it=   0
ccf, mink1, lambda=1, gamma=4, it= 100
ccf, mink1, lambda=1, gamma=4, it=1000
ccf, mink1, lambda=1, gamma=4, it=5000
c0f, mink1, lambda=1, gamma=4, it=   0
c0f, mink1, lambda=1, gamma=4, it= 100
c0f, mink1, lambda=1, gamma=4, it=1000
c0f, mink1, lambda=1, gamma=4, it=5000
0.

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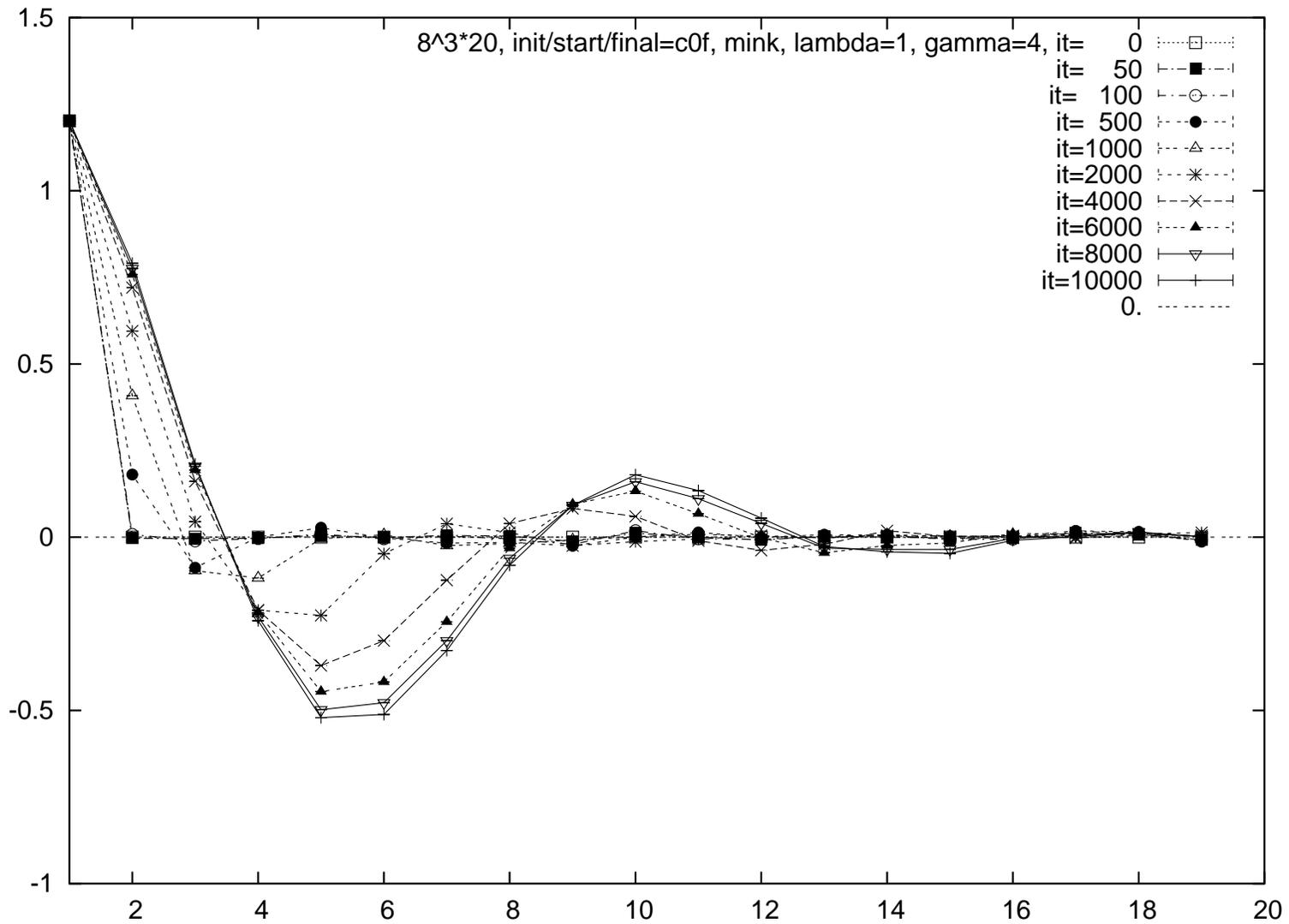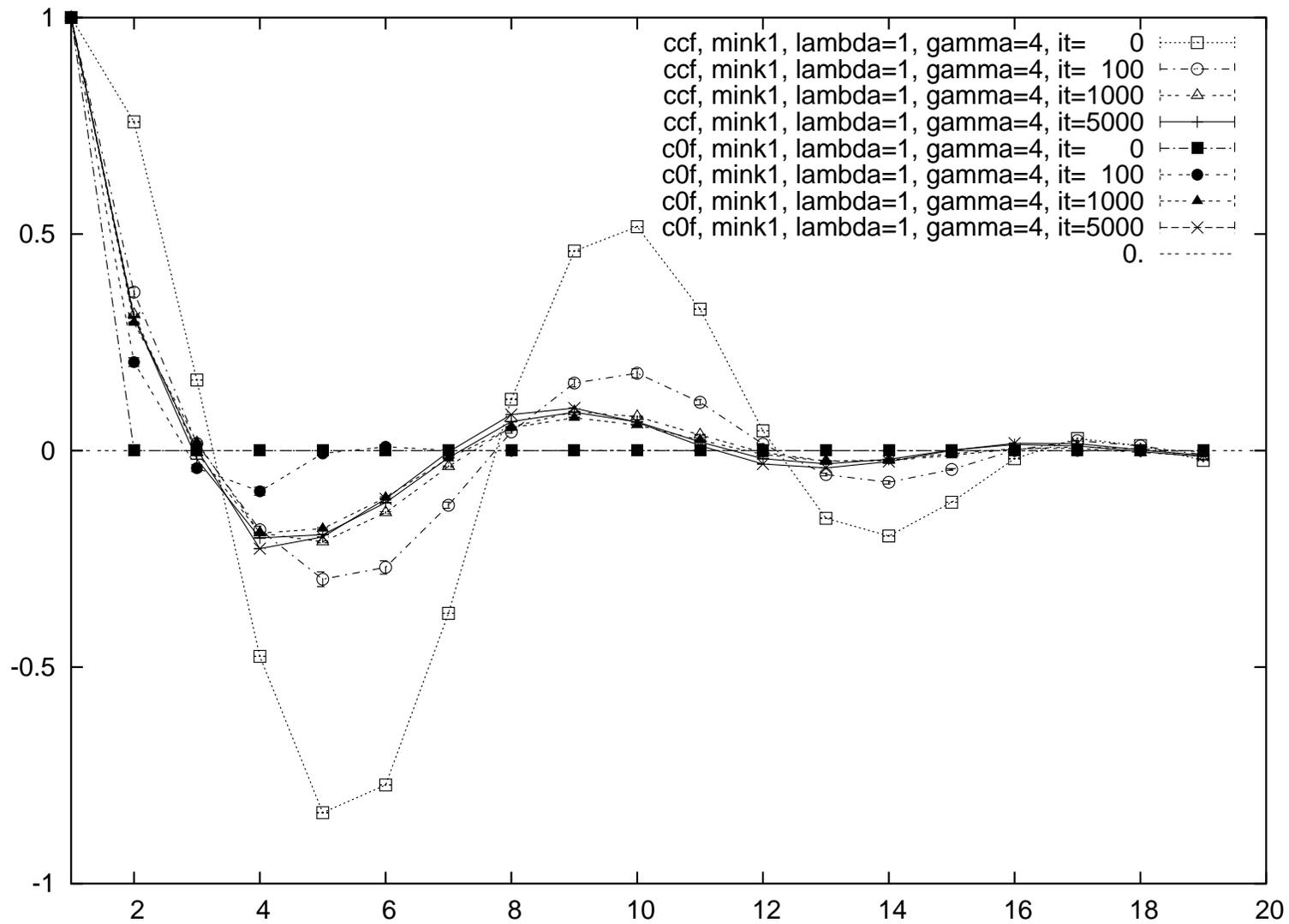
Figure 6: $ReC(t), M = 0, \lambda = 1$, vs ω, various t.
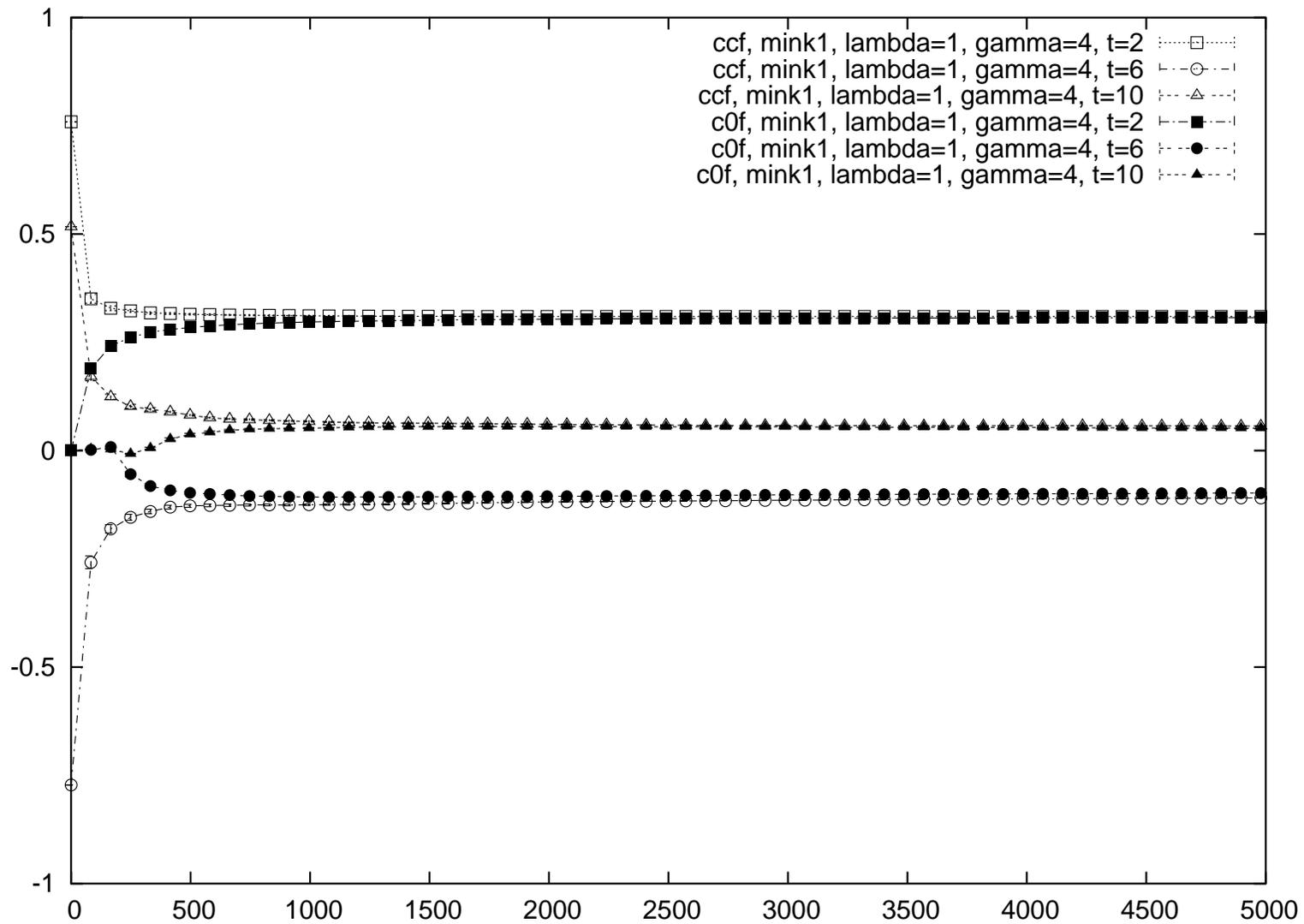
Figure 7: $ReC(t), M = 0, \lambda = 1$, vs ω, various t.
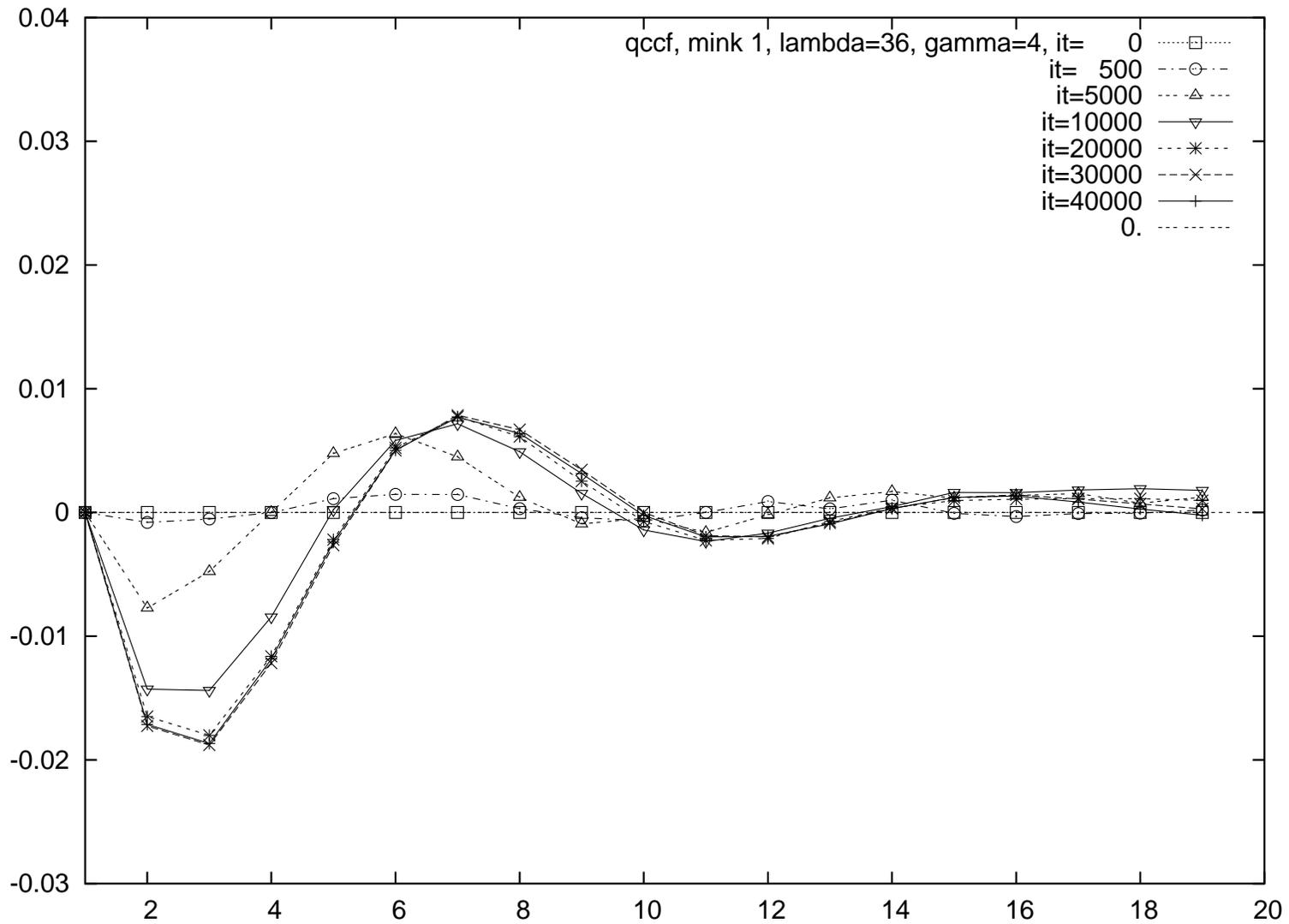
Figure 8: $ReC(t), M = 0, \lambda = 36.$
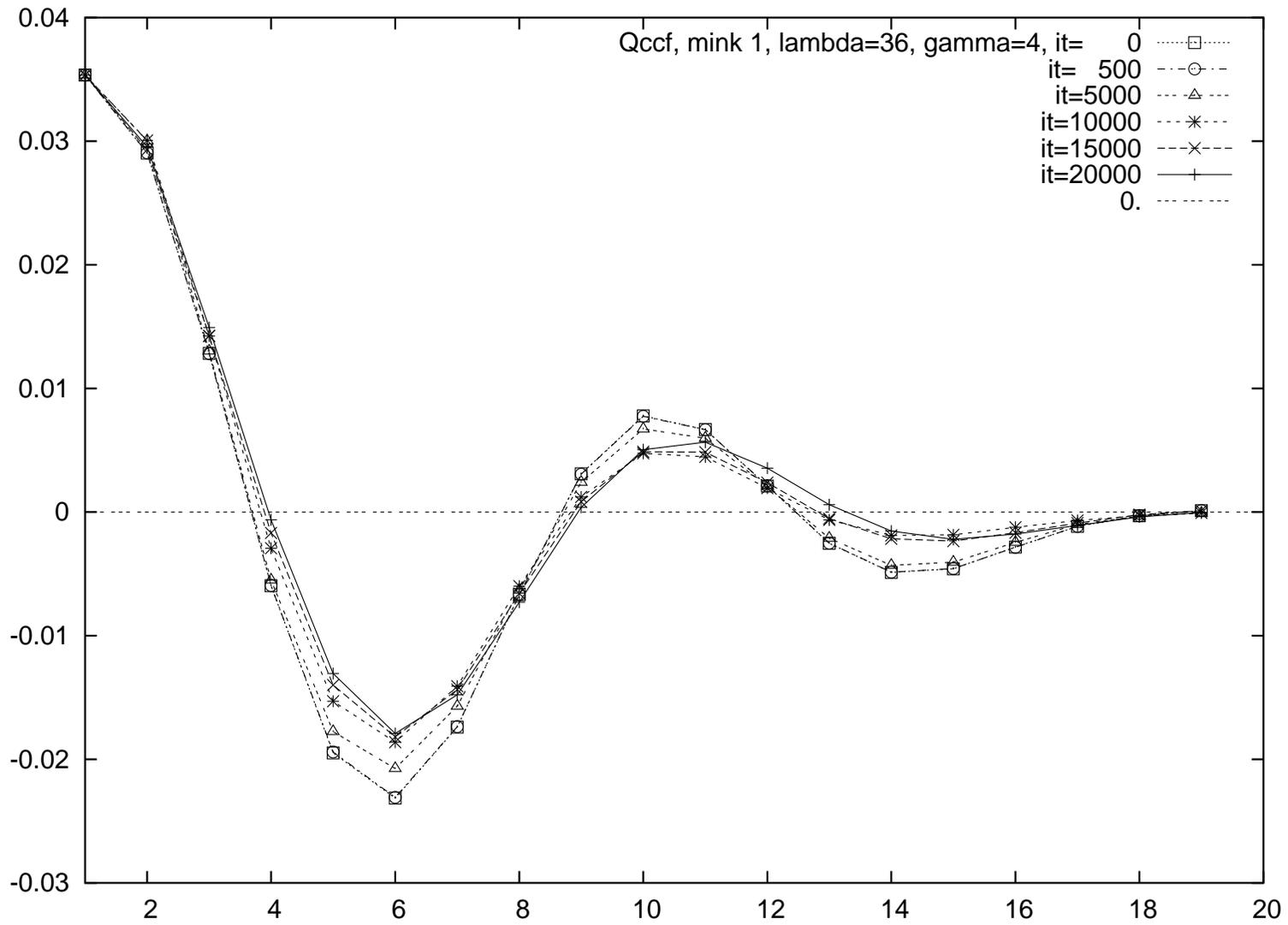
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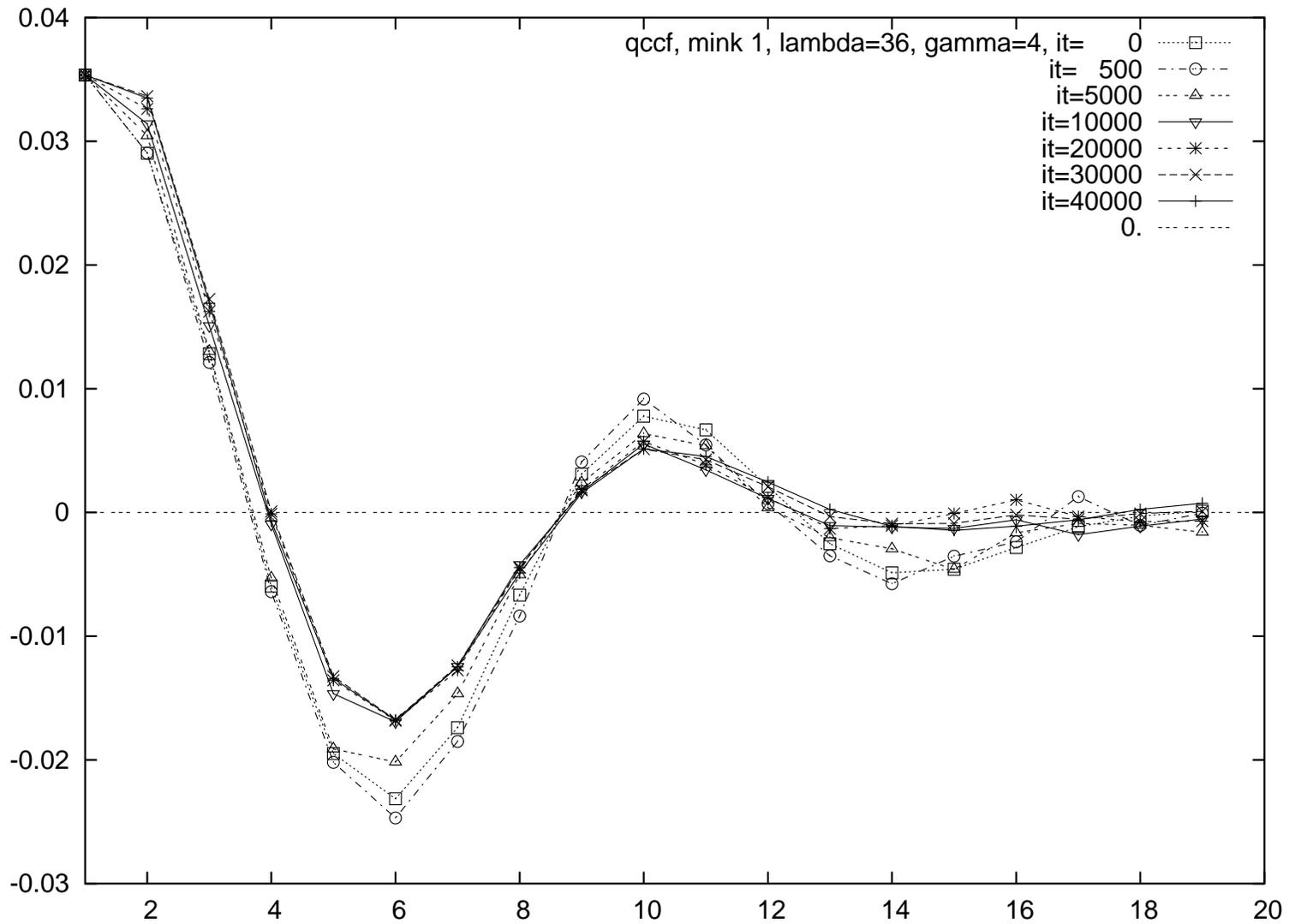
Figure 9: $ImC(t), M = 0, \lambda = 36.$
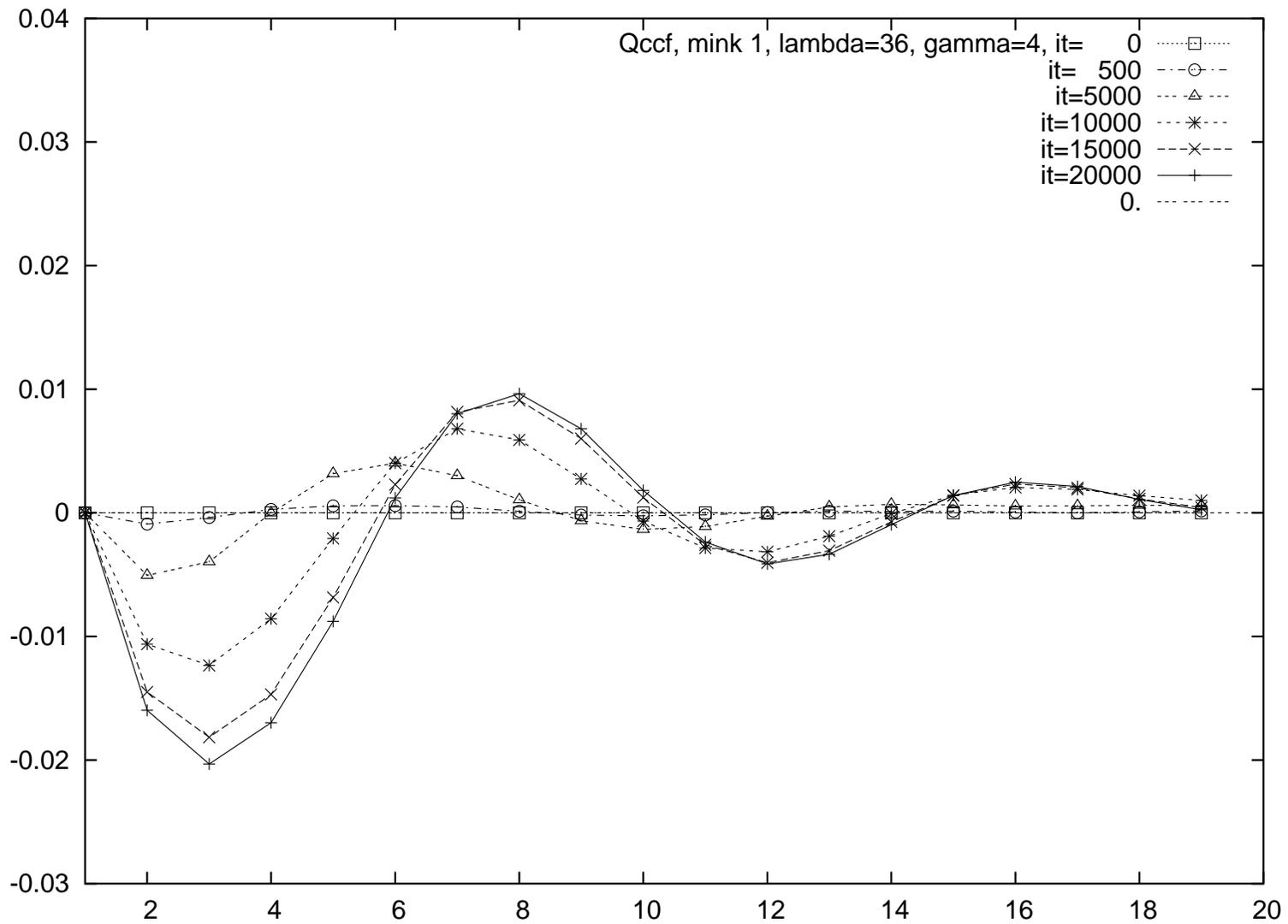
Figure 10: $ReC(t), M = 0, \lambda = 36$, 0 noise.
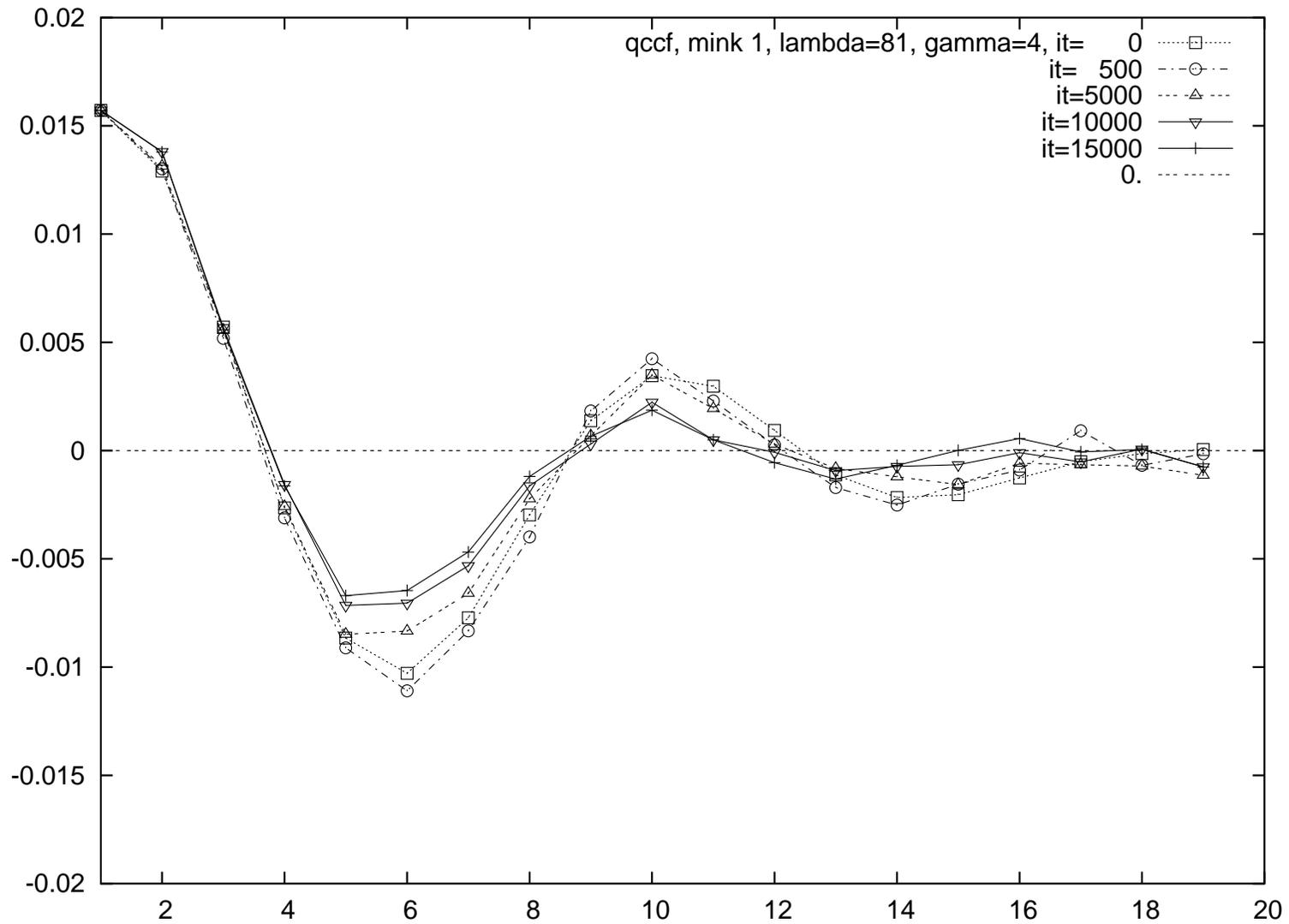
Figure 11: $ImC(t), M = 0, \lambda = 36$, 0 noise.
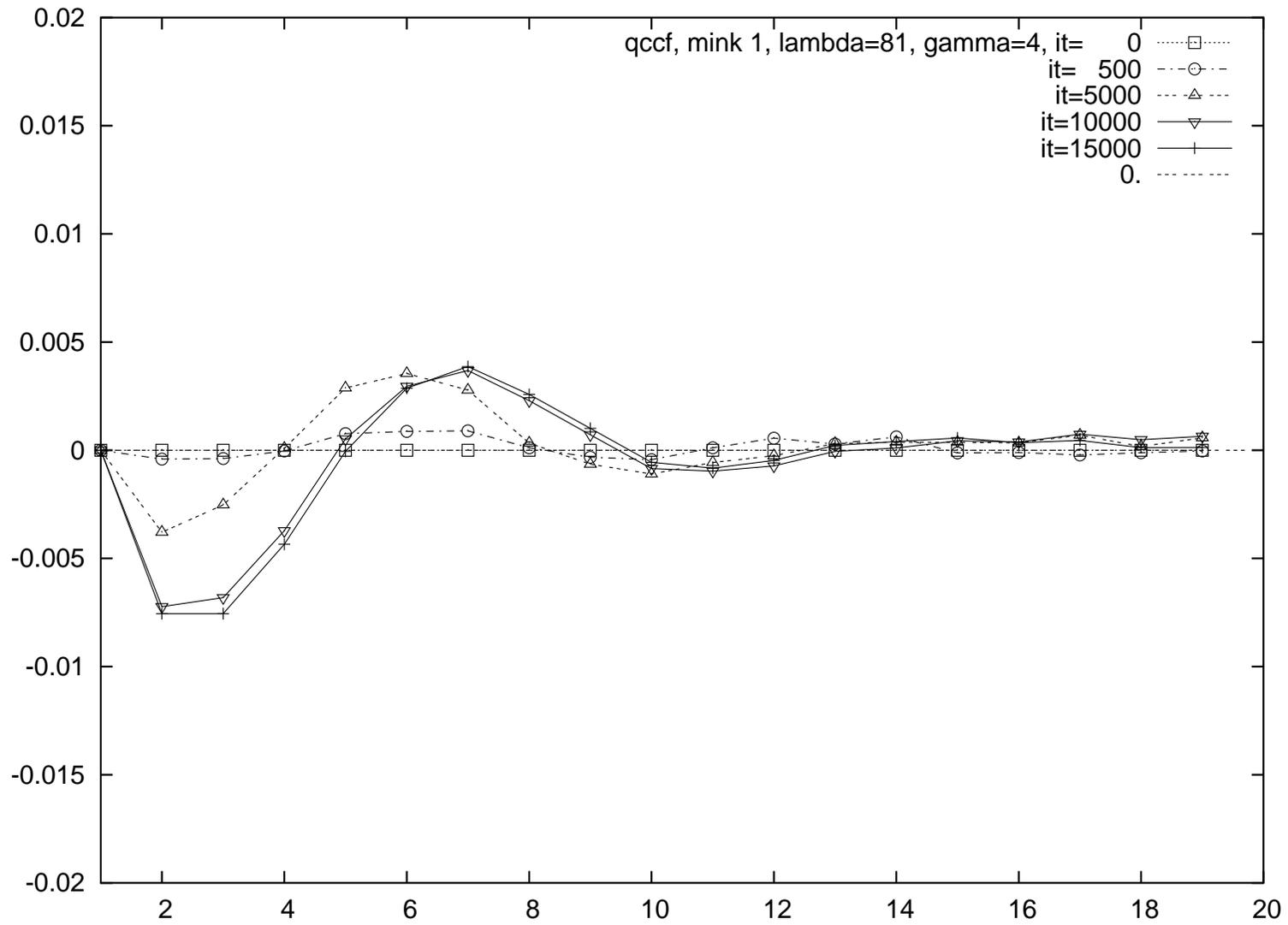
Figure 12: $ReC(t), M = 0, \lambda = 81$.

Figure 13: $ImC(t), M = 0, \lambda = 81.$

Outlook

- Complete the study of the ϕ^4 model:

  – Simulate systematically the model, compare with classical approach.

  – Study the "F-P" hamiltonian, convergence, "regularization", etc.

- Extend to Yang-Mills theory. Perspective for simulation:

  – Stochastic process in the Lie algebra, driving force obtained from the compact action

  – Possibly use stochastic gauge fixing