The Banks-Casher relation for $\mu_B \neq 0$

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WHAT ?

• Chiral symmetry breaking in the spectrum of the Dirac operator

WHY ?

• Improve our analytic understanding of chiral symmetry breaking

• Non trivial because of the sign problem

HOW ?

• **Input:** Exact solution for the spectrum of the QCD Dirac operator at $\mu_B \neq 0$ in the ϵ-regime

• A complex eigenvalue density in the complex plane due to the sign problem

Banks Casher at $\mu_B = 0$

$$\begin{aligned}
\langle\bar{\psi}\psi\rangle(m) &= \frac{1}{V}\partial_m \log Z(m) \\
&= \frac{1}{V}\left\langle \mathrm{Tr}\frac{1}{D_\eta\gamma_\eta + m}\right\rangle \\
&= \frac{1}{V}\left\langle \sum_j \frac{1}{z_j + m}\right\rangle \\
&= \frac{1}{V}\int dy\, \rho(y)\frac{2m}{y^2+m^2}
\end{aligned}$$

$$\langle\bar{\psi}\psi\rangle = \lim_{m\to 0}\lim_{V\to\infty}\langle\bar{\psi}\psi\rangle(m)$$

$$\langle\bar{\psi}\psi\rangle = \frac{\pi}{V}\rho(0)$$

Banks Casher NPB 169 (1980) 103

Definition of the eigenvalue density for $\mu \neq 0$

Eigenvalue equation

$$(D_\eta \gamma_\eta + \mu \gamma_0)\psi_j = z_j \psi_j$$

Eigenvalue density

$$\rho(z) \;\equiv\; \left\langle \sum_j \delta^2(z - z_j) \right\rangle_{\mathrm{QCD}}$$

$$\langle \mathcal{O} \rangle_{\mathrm{QCD}} \equiv \frac{\int \mathrm{d}A \;\mathcal{O}\,\mathrm{det}(D_\eta \gamma_\eta + \mu \gamma_0 + m)^{N_f}\; \mathrm{e}^{-S_{\mathrm{YM}}(A)}}{\int \mathrm{d}A\,\mathrm{det}(D_\eta \gamma_\eta + \mu \gamma_0 + m_f)^{N_f}\; \mathrm{e}^{-S_{\mathrm{YM}}(A)}}$$

Quenched $\mu \neq 0$

The chiral condensate from the eigenvalue density

$$\begin{aligned} \langle \bar{\psi}\psi \rangle(m) &= \frac{1}{V}\partial_m \log Z(m;\mu) \\ &= \frac{1}{V}\int d^2z\,\rho(z)\frac{2m}{-z^2+m^2} \end{aligned}$$

No δ-function as $m \to 0$: $\langle \bar{\psi}\psi \rangle = 0$

Chiral symmetry breaking
trough pion condensate

Stephanov PRL 76 (1996) 4472
Toublan Verbaarschot Int.J.Mod.Phys. B15 (2001) 1404
Hands Montvay Morrison Oevers Scorzato Skullerud, EPJ C17 (2000) 285

The eigenvalue density in the ϵ-regime

Uniquely determined by chiral symmetry
$SU(N_f) \times SU(N_f) \to SU(N_f)$

Structure

$$\rho(z) \equiv \rho_{N_f=0}(z) + \rho_{corr}(z)$$

Where $\rho_{corr}(z)$ is complex and oscillating

1) Envelope $\sim \exp(\pm V)$

2) Period $\sim \frac{1}{V}$

Osborn PRL 93 (2004) 222001
Akemann Osborn Splittorff Verbaarschot NPB 712 (2005) 287

Unquenched $\mu \neq 0$

The chiral condensate from the eigenvalue density

$$\begin{aligned}
\langle \bar{\psi}\psi \rangle(m) &= \frac{1}{V}\partial_m \log Z(m;\mu) \\
&= \frac{1}{V}\int dxdy\, \rho(x,y)\frac{1}{x+iy+m}
\end{aligned}$$

The oscillations of the density are responsible for chiral symmetry breaking

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Do the y-integral first

Structure:

$$\rho_{corr} = e^{-V[y^2+f(x,m;\mu)]} e^{iVy\,g(x,m;\mu)}$$

In the $y = a + ib$-plane

$$\begin{aligned}\rho_{corr} \quad = \quad & e^{-V[a^2-b^2+f(x,m;\mu)]-Vb\,g(x,m;\mu)} \\ & \times e^{iVa\,g(x,m;\mu)-iV2ab}\end{aligned}$$

For $m < x < 2\mu^2 F_\pi^2 / \langle \bar{\psi}\psi \rangle$**:**

ρ_{corr} suppressed in strip to the left of the pole.

Deform the contour into this strip

$\Rightarrow \int dy \frac{\rho_{corr}}{x+iy+m}$ is given by the pole.

Osborn, Splittorff, Verbaarschot, PRL 94 (2005) 202001

The contribution from the pole

The residue at the pole is

$$\rho_{corr}(z=m) = -\frac{V\langle\bar{\psi}\psi\rangle^2}{4\pi\mu^2}$$

independent of x simply because

$$\rho(z=m) = 0$$

and

$$\rho(z) \equiv \rho_{N_f=0}(z) + \rho_{corr}(z)$$

μ-independent chiral condensate from a complex & oscillating μ-dependent density

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Conclusion

- An oscillatory spectral density can give a discontinuity of the chiral condensate.
- The entire oscillating region of the complex eigenvalue plane contributes to the chiral condensate
- Uncovered using: The exact eigenvalue density of the QCD Dirac operator for fixed

$$m\langle\bar{\psi}\psi\rangle V, \quad z\langle\bar{\psi}\psi\rangle V, \quad \mu_B^2 F_\pi^2 V$$

  which has oscillations with a period of order $1/V$ and an amplitude $\sim e^{\# V}$