The chiral and deconfinement phase transitions in dense two-colour QCD

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- Physics Overview
- The simulation
- Some preliminary results

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Motivation

Why 2-colour QCD?

No sign problem $\Rightarrow$ can simulate with standard algorithms.

Can use as test lab for real QCD!

Similarities

- Gluodynamics: little distinction between SU(2) and SU(3)?
- (Pseudo-)Critical line at high T, low μ?
- Deconfinement?
- Fate of vector meson?

Differences

- Baryons are bosons (diquarks)!

- q and $\bar{q}$ live in equivalent reps. of the color group, so chiral multiplets contain both $q\bar{q}$ mesons and qq baryons.

- For $\mu \geq \frac{1}{2}m_\pi$ a superfluid diquark condensate develops, together with a non-zero baryon density.

- No colour superconductivity!

Prior knowledge

If $m_\pi \ll m_\rho$ the behaviour as μ is varied can be studied using chiral perturbation theory (χPT)

Analytical prediction:

$$n_B \propto f_\pi^2(\mu - \mu_o); \quad \langle qq \rangle \propto \sqrt{1 - \left(\frac{\mu_0}{\mu}\right)^4} \Rightarrow \lim_{\mu\to\infty} = \text{const.}$$

confirmed by simulations with staggered fermions

Alternative picture: BCS condensation

$$n_B \propto \mu^3; \quad \langle qq \rangle \propto \Delta\mu^2; \quad \varepsilon \propto \mu^4$$

from counting states within the Fermi sphere.

Lattice Formulation

The $N_f = 2$ action we want to simulate is:

$$S = \bar{\psi}_1 M \psi_1 + \bar{\psi}_2 M \psi_2 - J \bar{\psi}_1 (C\gamma_5) \tau_2 \bar{\psi}_2^{tr} + \bar{J} \psi_2^{tr} (C\gamma_5) \tau_2 \psi_1$$

and includes scalar isoscalar diquark source terms

$$C\gamma_\mu C^{-1} = \gamma_\mu^* \Rightarrow C^\dagger = C^{-1} = -C, \ [C, \gamma_5] = 0$$

$M(\mu)$ is the textbook Wilson fermion operator $\Rightarrow$

$$\begin{aligned} \gamma_5 M(\mu) \gamma_5 &= M^\dagger(-\mu) && (3) \\ \tau_2 U_\mu \tau_2 = U_\mu^* \quad \Rightarrow \quad C\tau_2 M(\mu) C^{-1} \tau_2 &= M^{tr}(-\mu) && (4) \\ \Rightarrow \quad (C\gamma_5)\tau_2 M(\mu) (C\gamma_5)^{-1} \tau_2 &= M^*(\mu) && (5) \end{aligned}$$

Property (5) implies $\det M(\mu)$ is real

With change of variables $\bar{\phi} = -\psi_2^{tr} C \tau_2$, $\phi = C^{-1} \tau_2 \bar{\psi}_2^{tr}$, the action can be rewritten

$$S = (\bar{\psi}, \bar{\phi}) \left(\begin{array}{cc} M(\mu) & J\gamma_5 \\ -\bar{J}\gamma_5 & M(-\mu) \end{array} \right) \begin{pmatrix} \psi \\ \phi \end{pmatrix} \equiv \bar{\Psi} \mathcal{M} \Psi$$

Since the action is bilinear, Grassmann integration yields det$\mathcal{M}$ rather than a Pfaffian. Using (1,2) we deduce

$$\mathrm{det}\mathcal{M} = \mathrm{det}(M^\dagger(\mu) M(\mu) + J\bar{J})$$

Hence $\bar{J} = J^*$ ensures det$\mathcal{M}$ is real and positive definite

Note that $N_f = 2$ ensures theory is asymptotically free for all couplings, with a controllable continuum limit, and is confining for $\mu = T = 0$

“u/d Partitioning”

It is easy to write HMC algorithm with weight $\mathrm{det}\mathcal{M}^\dagger\mathcal{M}$

$\Leftrightarrow N_f = 4$ flavors. However, it can be shown that

$$\mathrm{det}\mathcal{M}^\dagger\mathcal{M} \equiv [\mathrm{det}(M^\dagger(\mu)M(\mu) + \bar{J}J)]^2$$

$\Rightarrow$ permits pseudofermion action $\theta^\dagger(M^\dagger M + |J|^2)^{-1}\theta$

- requires matrix/vector operations of standard size

- permits Hamiltonian evaluation and hence an HMC algorithm describing $N_f = 2$ flavors

Simulation Details

Our initial runs use a $8^3 \times 16$ lattice with parameters

$\beta = 1.7$, $\kappa = 0.1780$

$\Rightarrow$ $a = 0.220\,\mathrm{fm}$, $m_\pi a = 0.800(2)$, $m_\pi / m_\rho = 0.920(3)$

$\Rightarrow$ χPT predicts onset of superfluid phase at $\mu_o a \simeq 0.4$

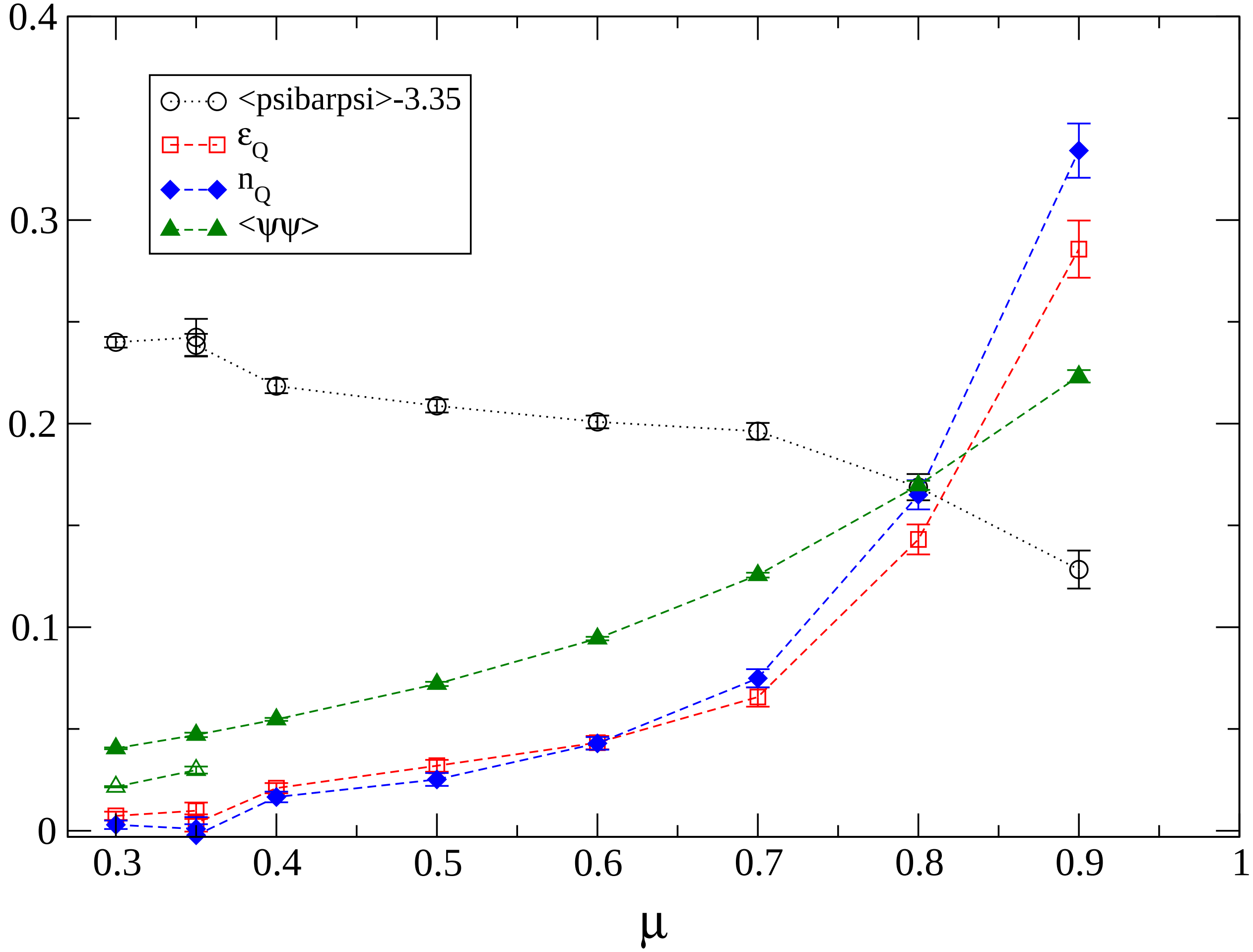
To counter IR fluctuations in the superfluid phase we use a diquark source $j = \kappa^{-1} J = 0.04$

Statistics so far:

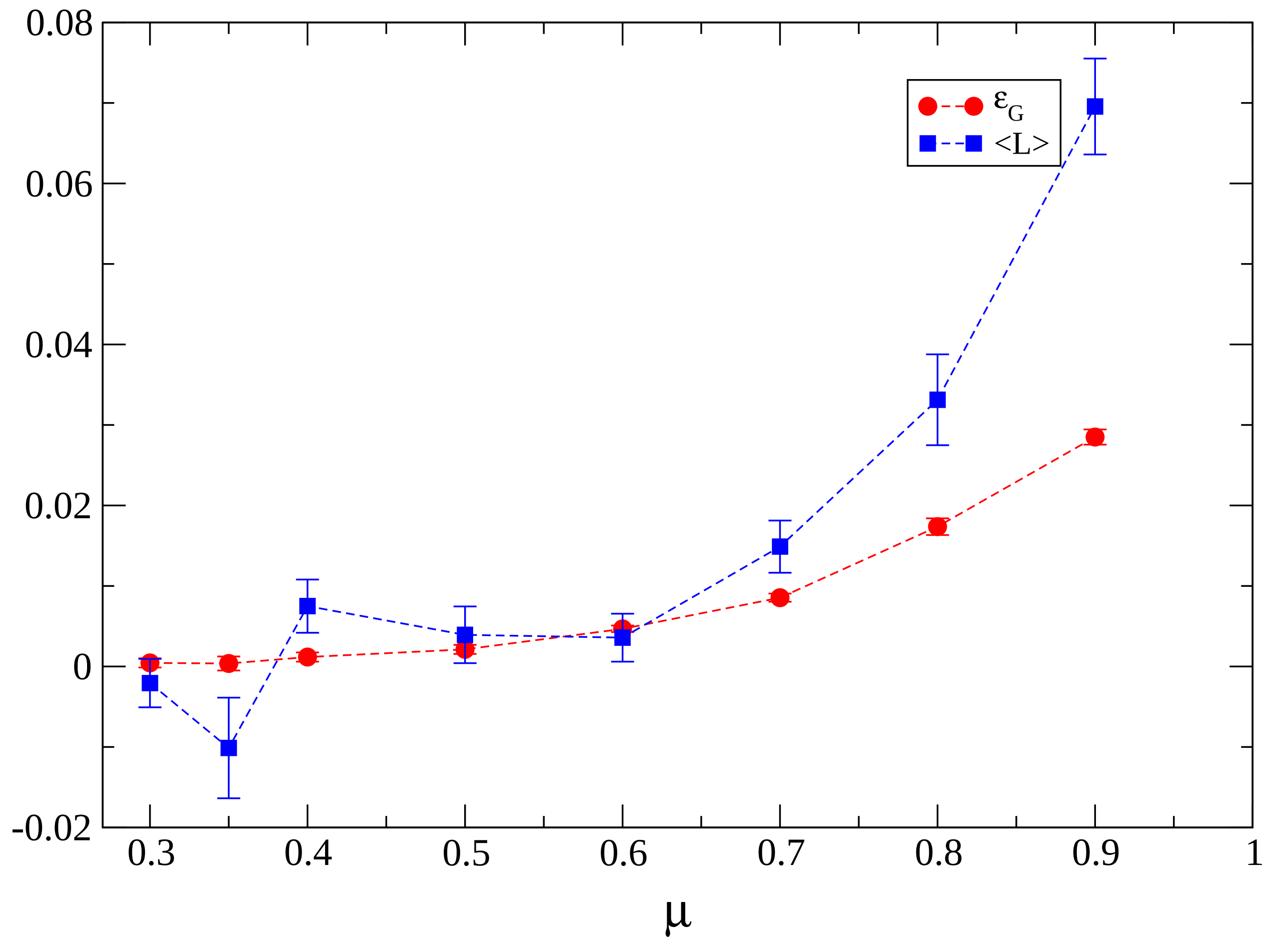
μ	N_{traj}	$\langle \ell \rangle$
0.3	190	1.0
0.35	45	1.0
0.4	302	0.5
0.4	90	1.0
0.5	42	1.0
0.5	304	0.5
0.6	108	1.0
	303	0.5
0.7	48	1.0
	302	0.5
0.8	58	1.0
0.9	120	0.5

The results for $n_B(\mu)$ suggest that f_π is much smaller for our system than for those studied previously with staggered fermions $\Leftrightarrow$ the effective boson degrees of freedom are more strongly interacting

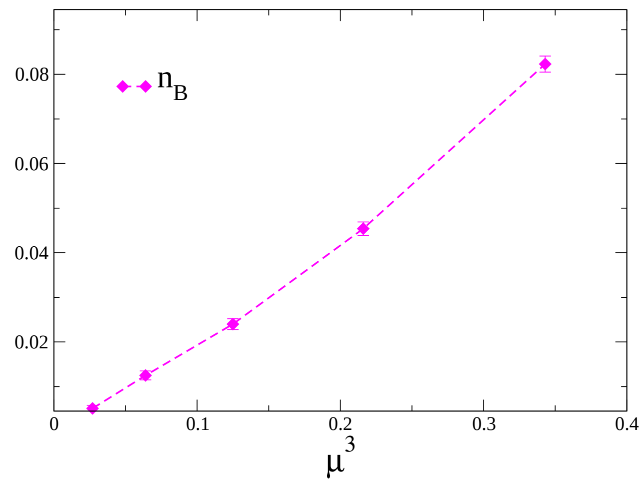
Quark Observables

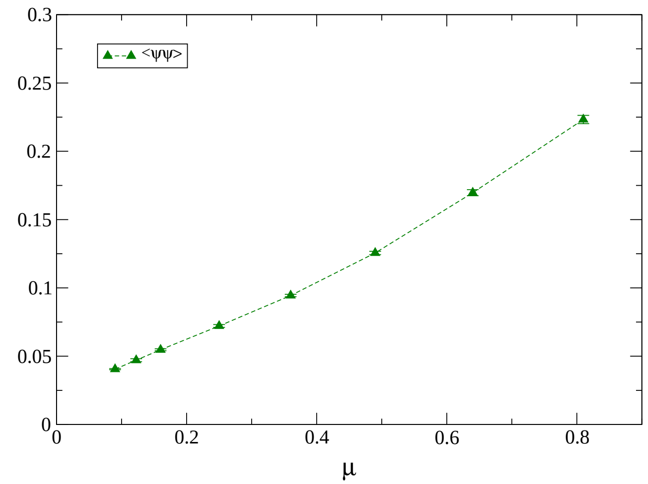


Gluonic Observables



Comparison with Fermi Surface Model

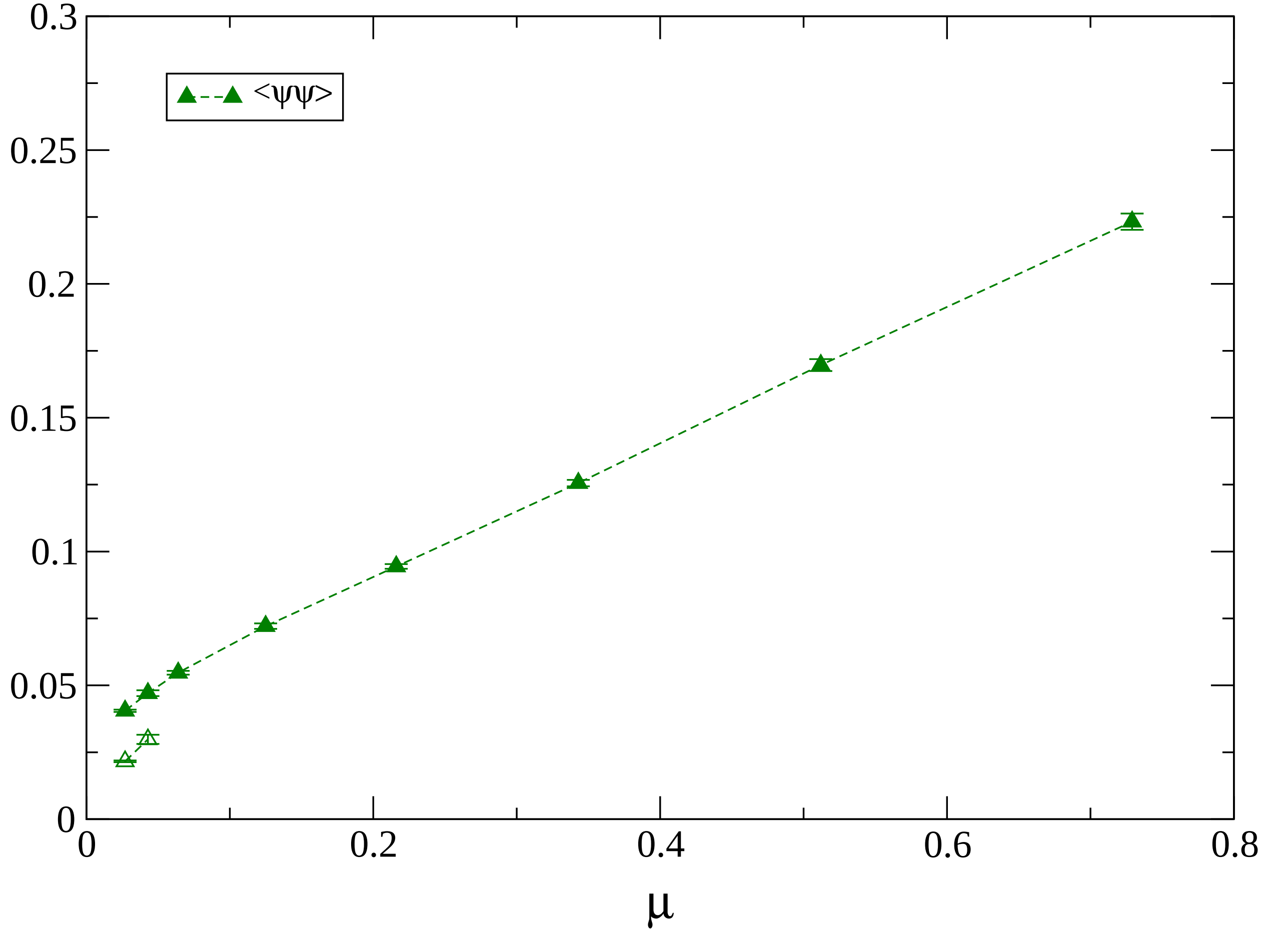




- $n_B \propto \mu^3$ for small n_B, with lattice artefacts beginning to show for larger μ?

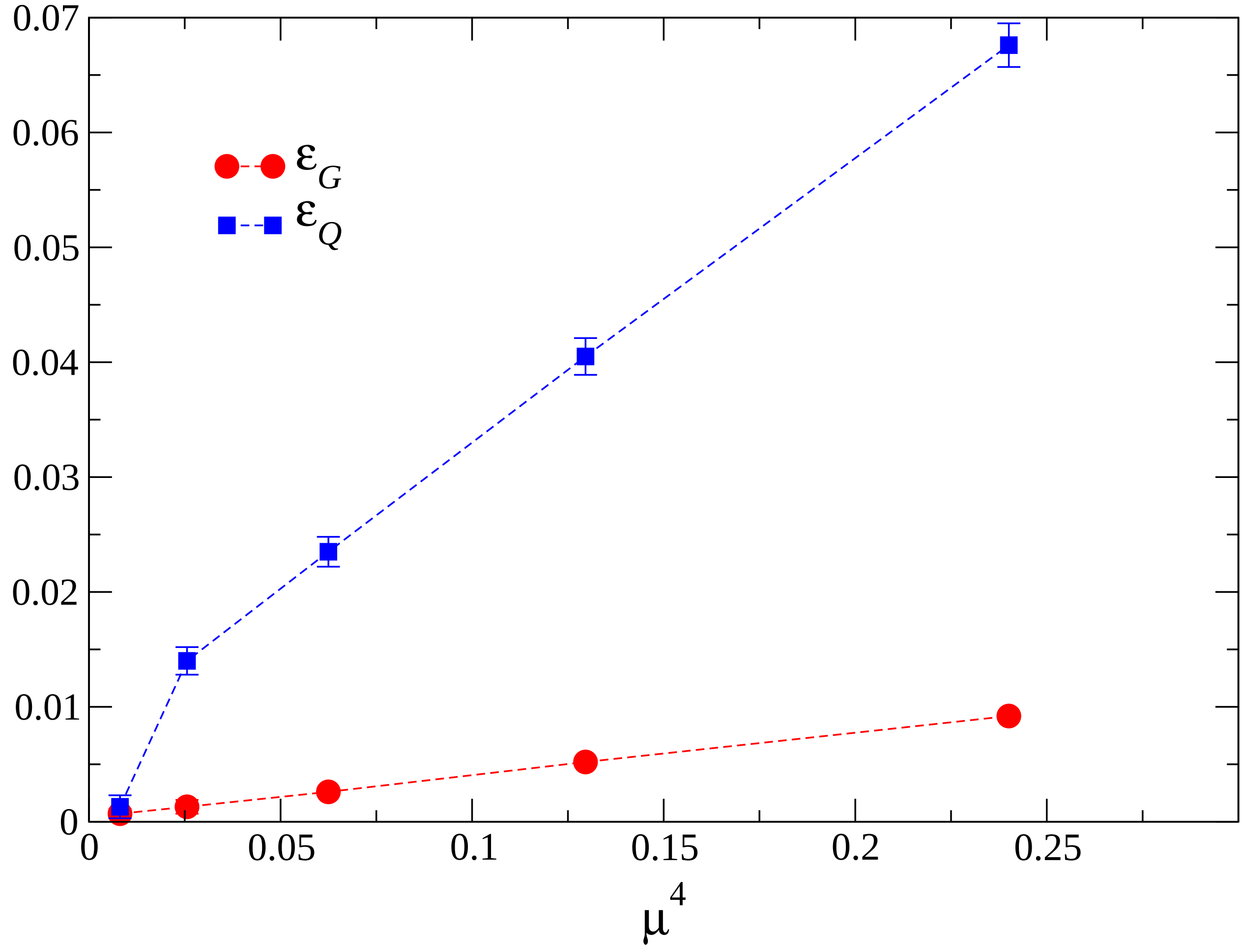
- $\langle qq \rangle \approx A + B\mu^2 \Rightarrow$ recover BCS behaviour as $j \to 0$?

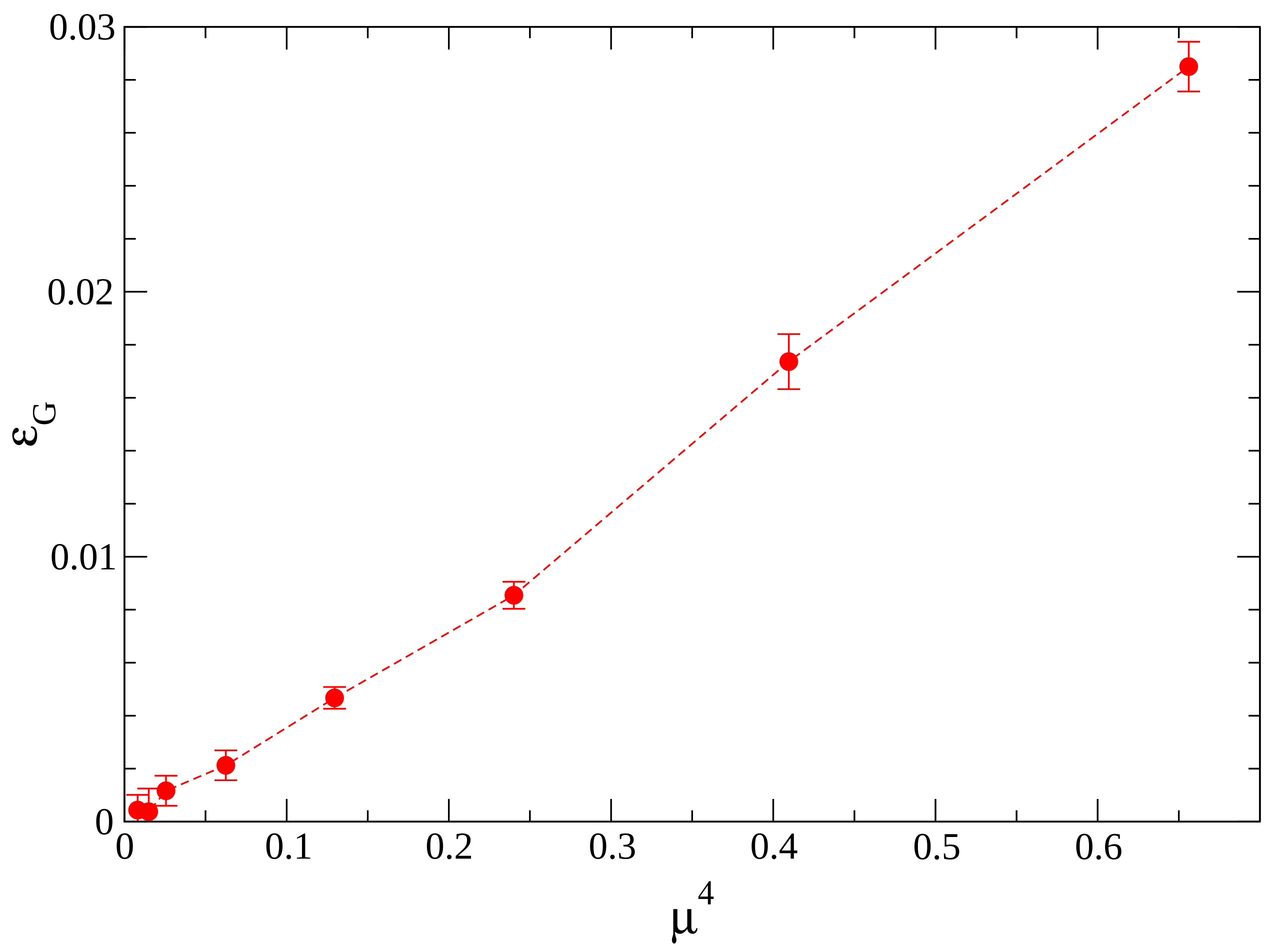
- Energy densities $\varepsilon_Q, \varepsilon_G \propto \mu^4$, with $\varepsilon_Q > \varepsilon_G$



0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
μ
<ψψ>

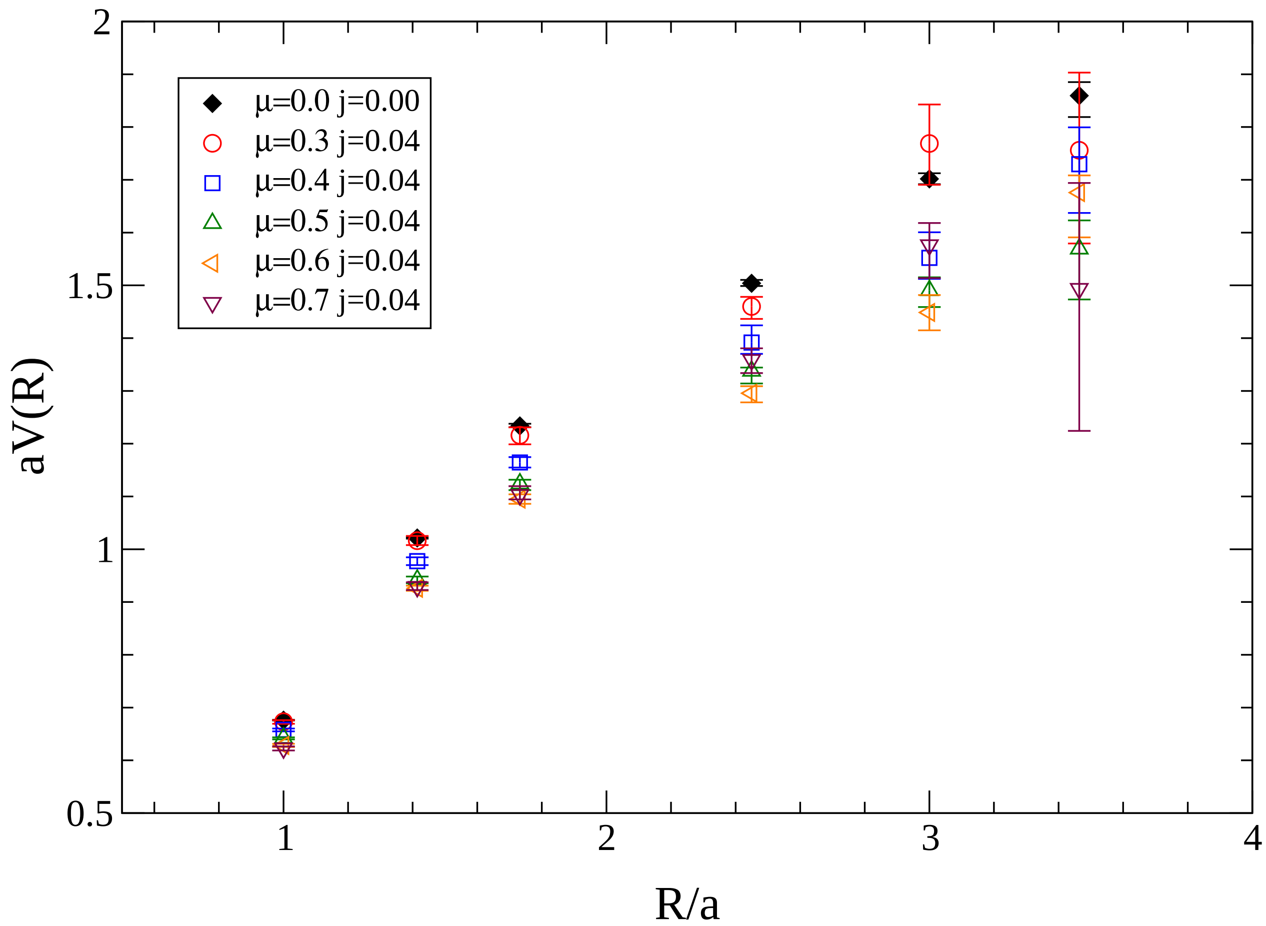
Energy Densities

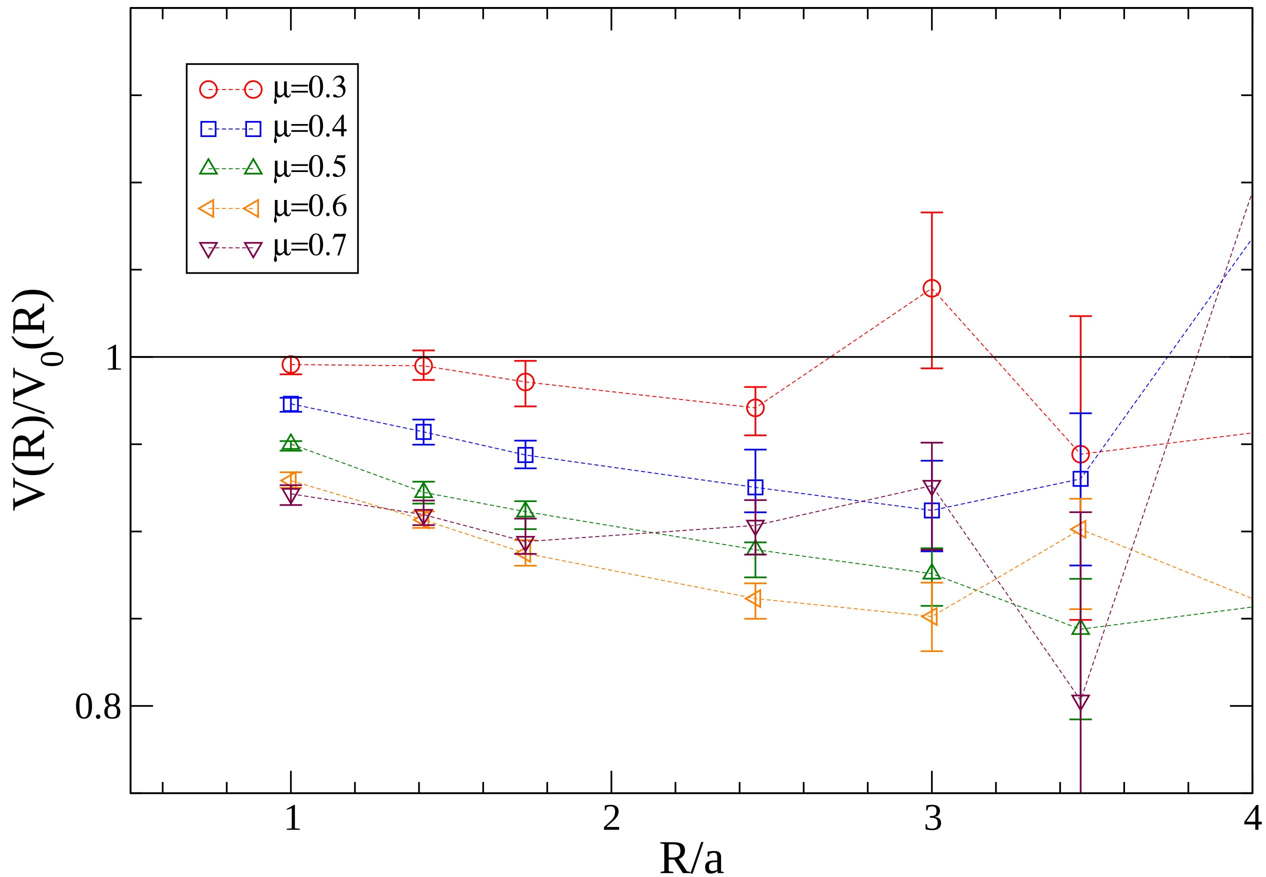




0.03
0.02
0.01
0
ε_G
0
0.1
0.2
0.3
0.4
0.5
0.6
μ^4

Static Quark Potential





μ=0.3
μ=0.4
μ=0.5
μ=0.6
μ=0.7
V(R)/V0(R)
1
0.8
1
2
3
4
R/a

Summary & Outlook

- Indications of a Fermi surface disrupted by diquark Cooper pairing, rather than a BEC formed from tightly bound diquark scalars

- Evidence for a non-vanishing gluon energy density due solely to a background quark charge density

- Evidence for screening of static potential for $\mu > 0.3$

- Evidence for deconfinement for $\mu \gtrsim 0.7$ from Polyakov loop?

- Future analysis to include: gluon propagator, hadron spectrum...