Eigenvalue correlations in quenched QCD at finite density

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based on:
discussions and lattice data from T. Wettig
and discussions with J.J.M. Verbaarschot

Eigenvalue correlations in quenched QCD at finite density

- QCD Dirac operator with chemical potential
- Low energy effective theories
  - Chiral Lagrangian
  - Chiral Random Matrix Theory (RMT)
- Exact results for low energy eigenvalue correlations
- Comparison to lattice
  - Density fits for low energy constants
  - Thouless energy from correlations

QCD Dirac Eigenvalues (μ=0)

- QCD Dirac operator

$$D = D_\eta \gamma_\eta$$

- chiral symmetry

$$\{D, \gamma_5\} = 0$$

- eigenvalues

$$D \psi_k = \quad i \lambda_k \psi_k$$

$$D \gamma_5 \psi_k = -i \lambda_k \gamma_5 \psi_k$$

Nonzero Quark Chemical Potential

- Grand Canonical Ensemble

$$Z = \sum_{N_B} e^{\mu N_B / T} Z_{N_B}$$

$$D(\mu) = D - \mu \gamma_0$$

- Complex eigenvalues

$$D(\mu)\psi_k = z_k \psi_k, \quad D(\mu)\gamma_5\psi_k = -z_k \gamma_5 \psi_k$$

- Sign problem

$$\det(D - \mu\gamma_0 + m) \in \mathbb{C}$$

Low Energy Effective Theories

- Zero-Momentum Chiral Lagrangian

$$\int dU \, \mathrm{e}^{\frac{1}{2}\Sigma V \,\mathrm{Tr}\, M (U + U^{-1}) - \frac{1}{4} F^2 V \,\mathrm{Tr}[\hat{\mu}, U][\hat{\mu}, U^{-1}]}$$

- Chiral Random Matrix Theory

$$\int dX \, dY \, w(X, Y) \; \prod_k \det \begin{pmatrix} m_k & i X + \mu_k Y \\ i X^H + \mu_k Y^H & m_k \end{pmatrix}$$

Eigenvalue Correlations at $\mu \neq 0$

- Gaussian Chiral RMT

$$w(X,Y) = e^{-\sigma N \mathrm{Tr}(X^H X) - \rho N \mathrm{Tr}(Y^H Y)}$$

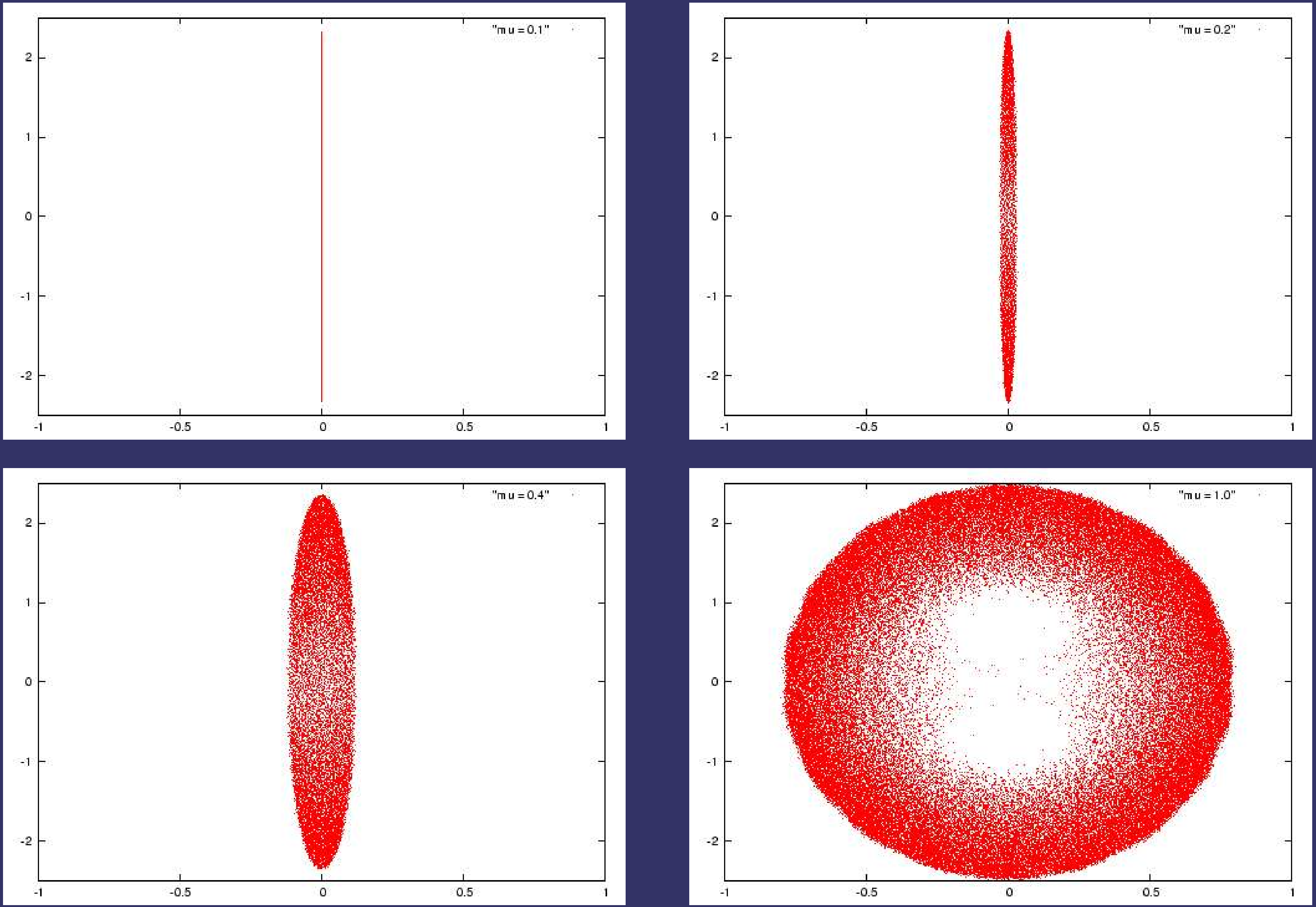
- Can solve exactly for all eigenvalue correlations (even for unquenched).

- Universal results in large N limit keeping $m_k \Sigma V$, $\mu_k^2 F^2 V$, (and $z\Sigma V$) fixed. (microscopic limit)

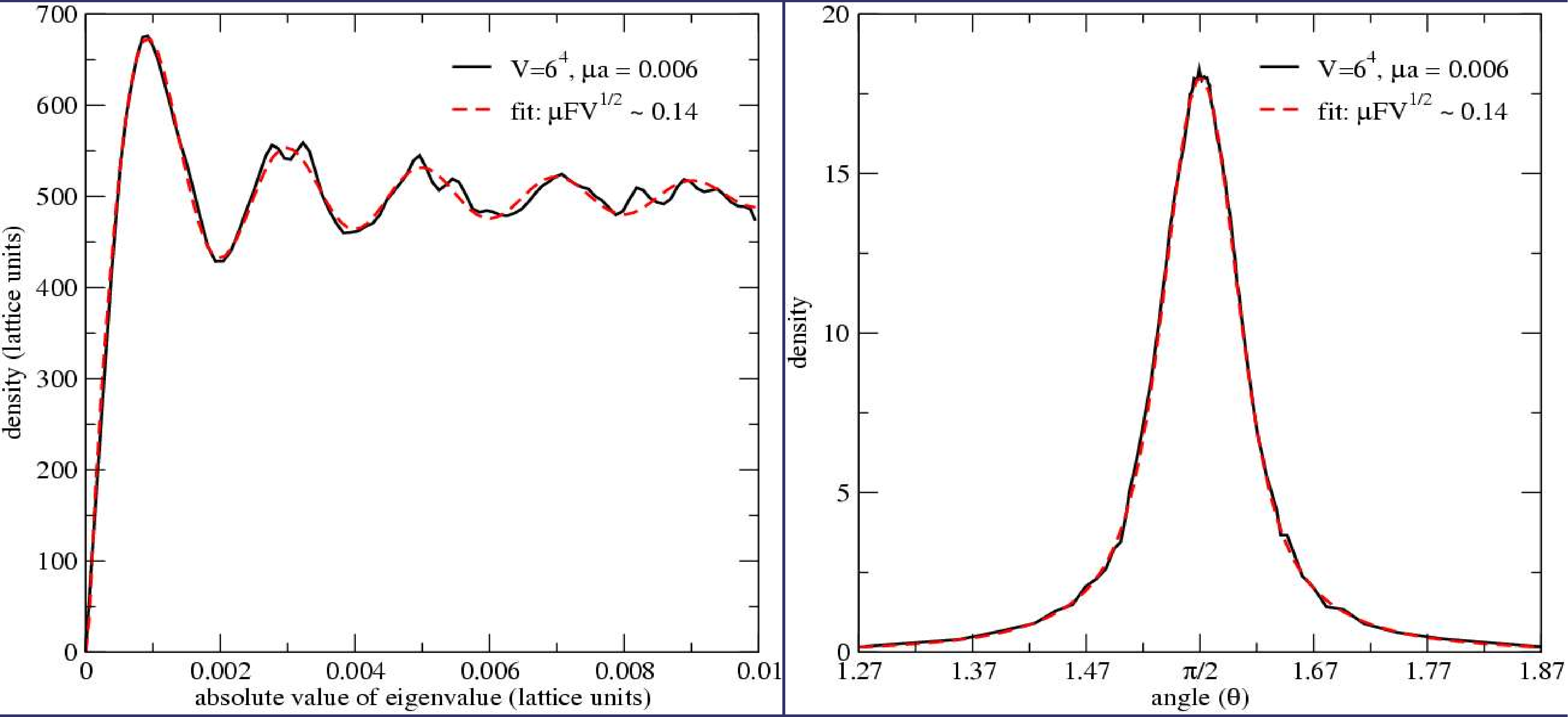
Lattices

- Quenched lattices at β=5.0, sizes: 6^4,8^4,10^4
- Staggered Dirac matrix
  - 6^4:
    μa = 0.006, 0.008, 0.03, 0.05, 0.1, 0.2, 0.4, 1
  - 8^4:
    μa = 0.00375, 0.2
  - 10^4:
    μa = 0.00216, 0.2

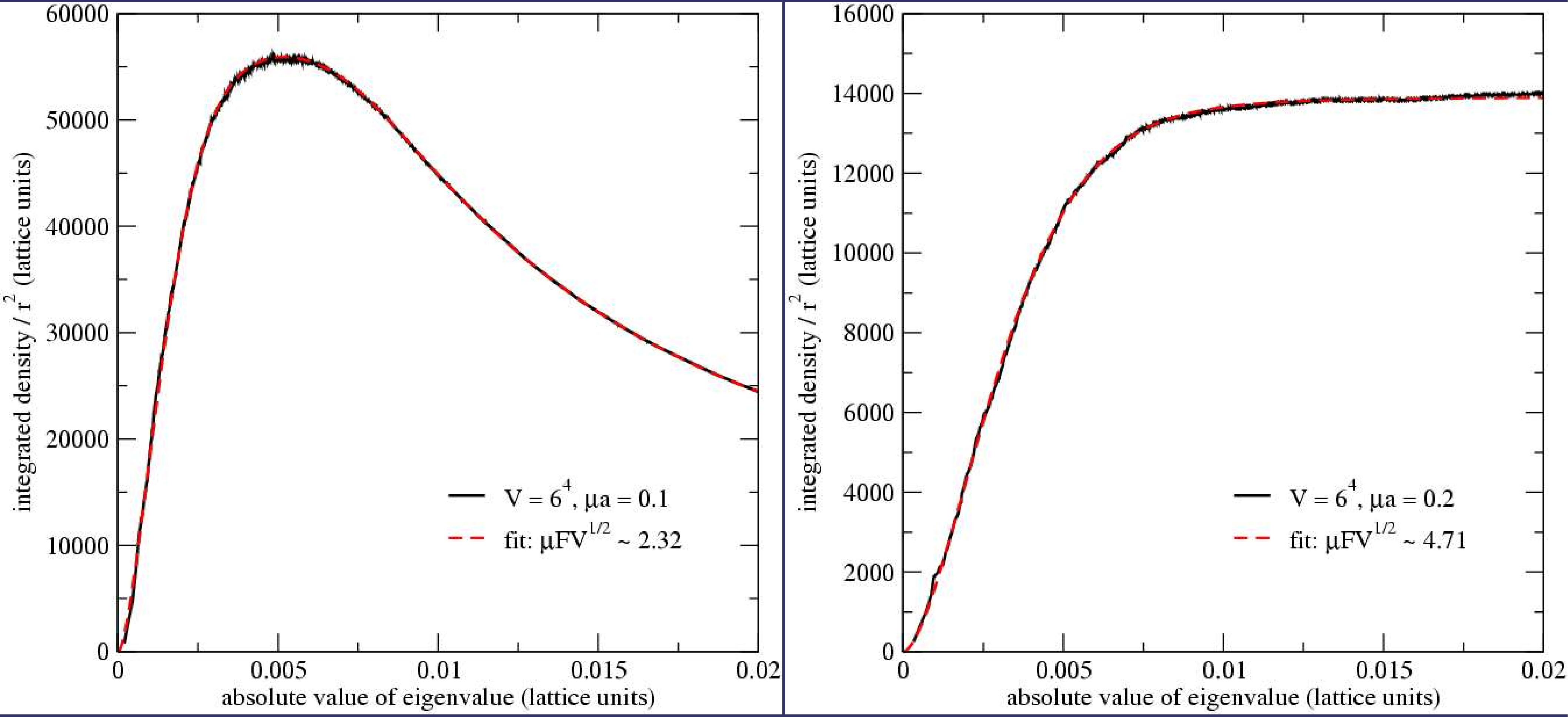
Macroscopic Eigenvalue Density (6^4)



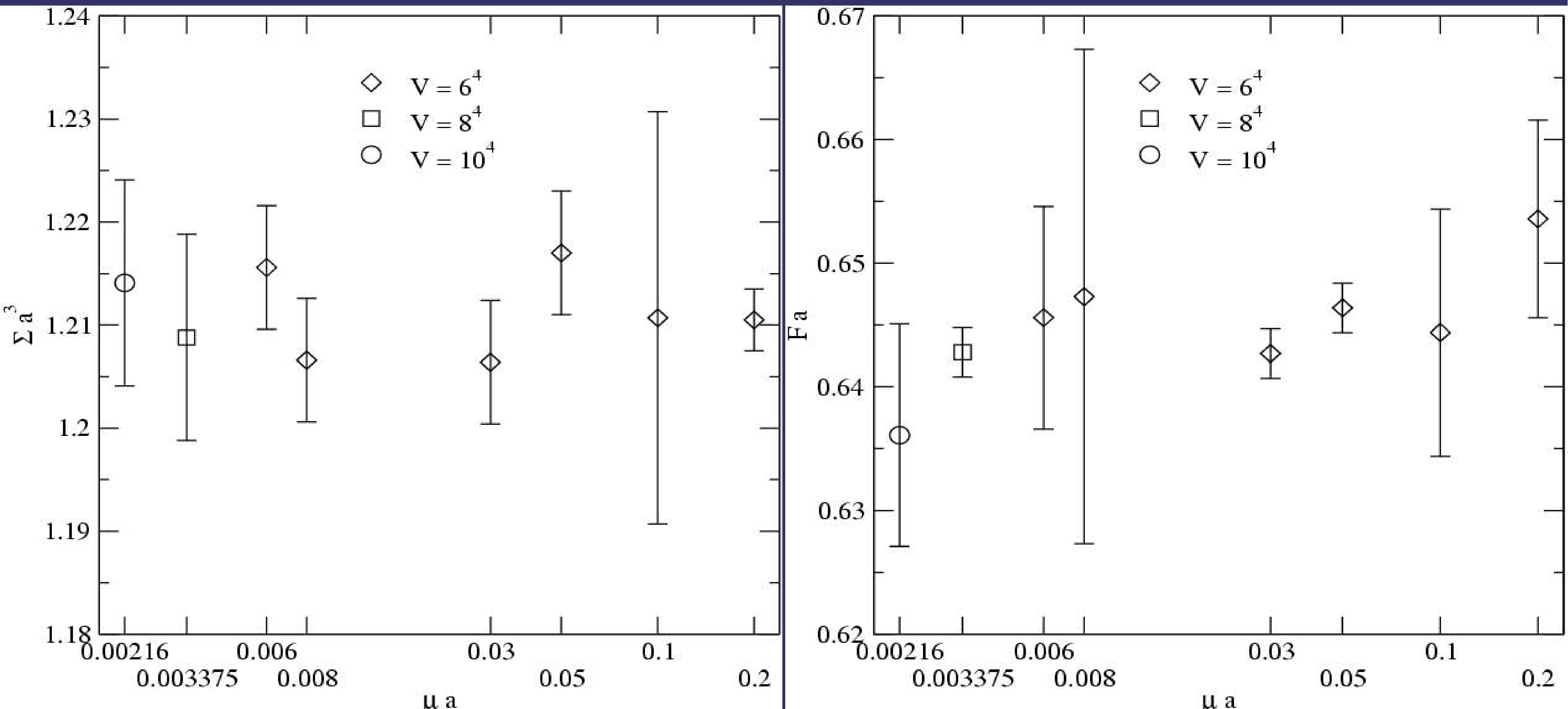
Microscopic Density



Radial Eigenvalue Density



Parameter Fits



Thouless Energy

- Universal results only at low energy.
- Zero-momentum approximation valid when (μ=0)

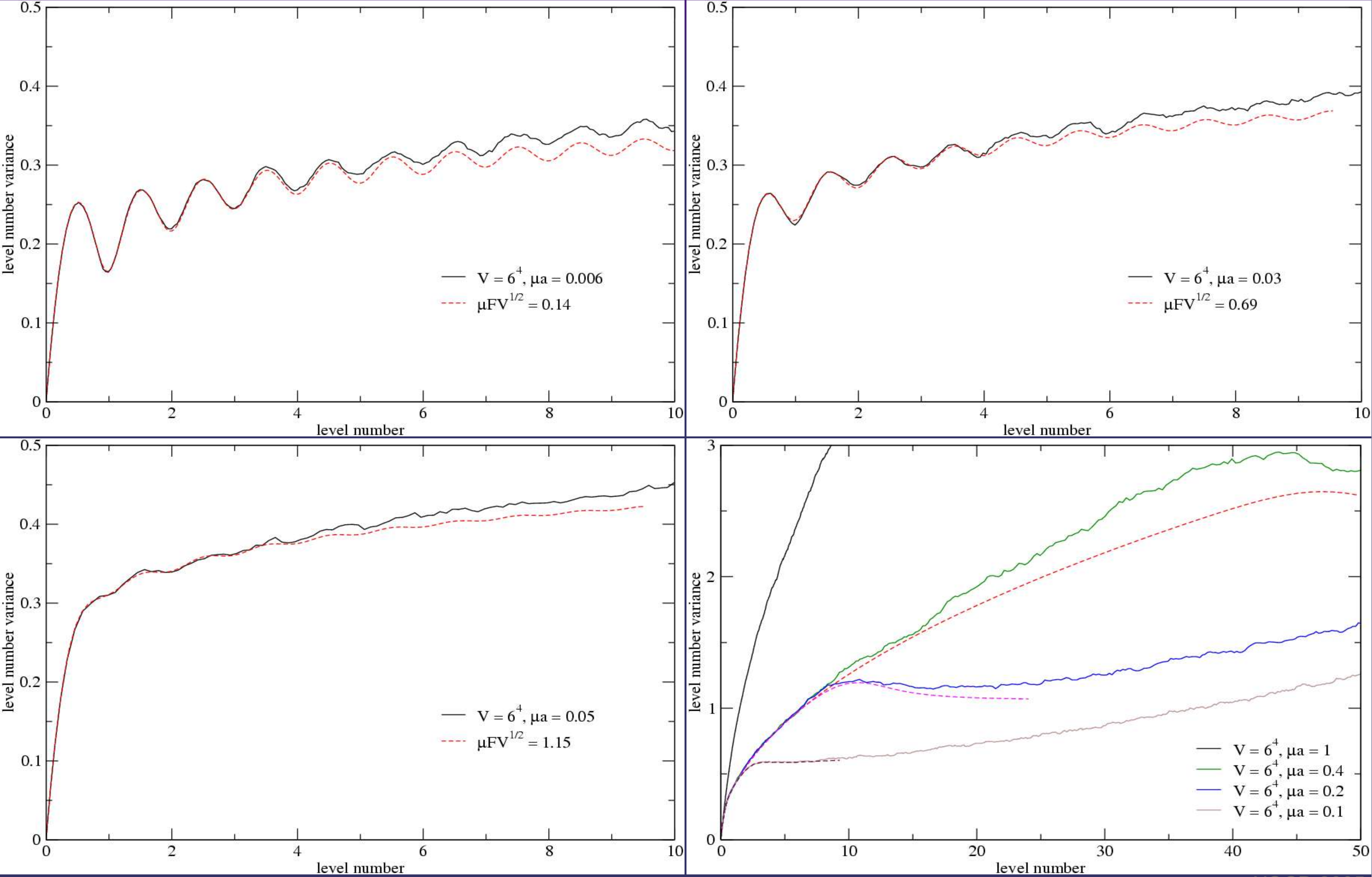
$$L \ll \frac{\pi}{m_\pi} \approx \frac{\pi}{\sqrt{2\Sigma|z|/F^2}}$$

$$|z| \ll \frac{\pi^2 F^2}{2\Sigma L^2} \simeq E_c$$

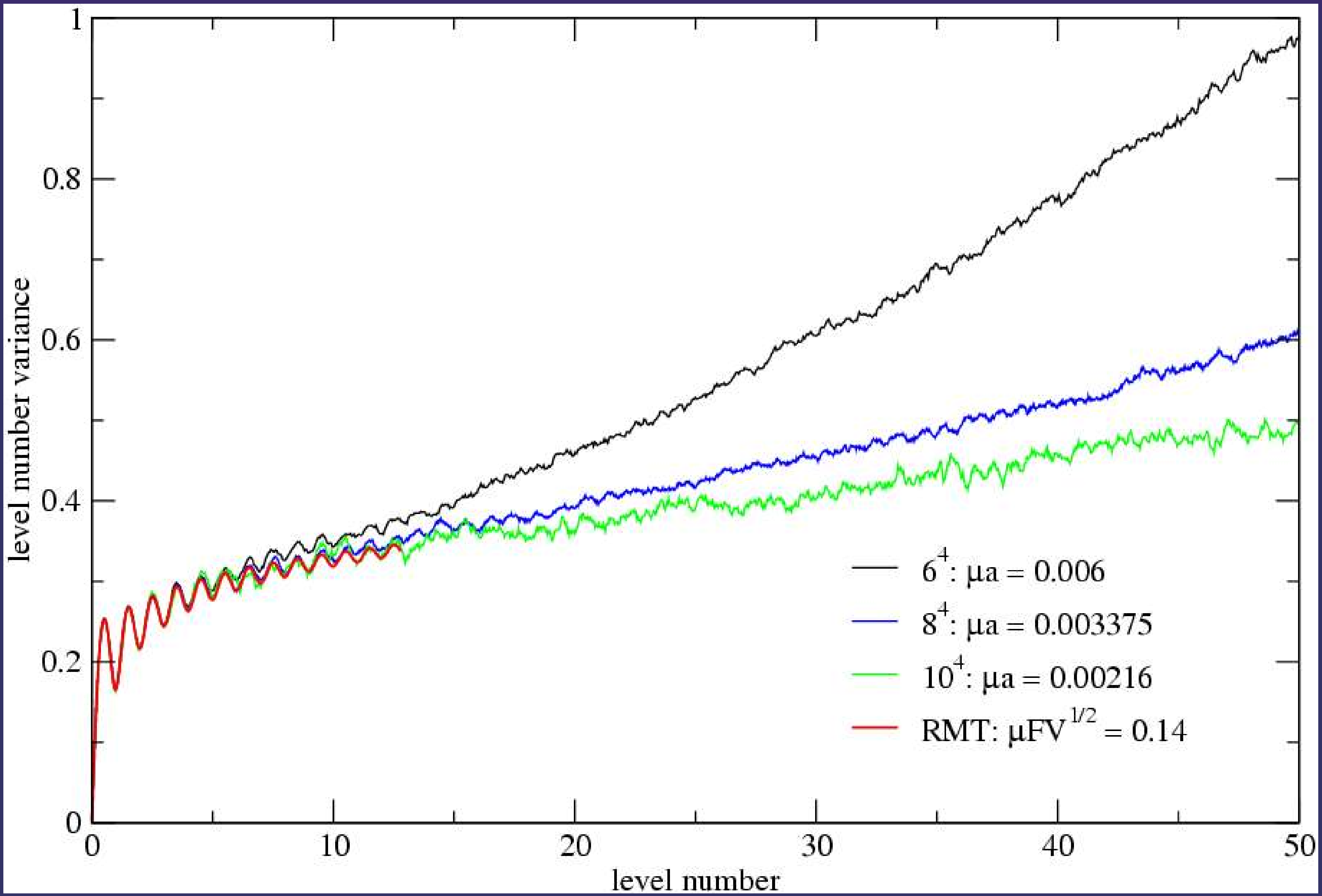
- In units of eigenvalue spacing (μ=0)

$$\frac{E_c}{\pi/\Sigma V} \simeq \frac{\pi}{2} F^2 L^2$$

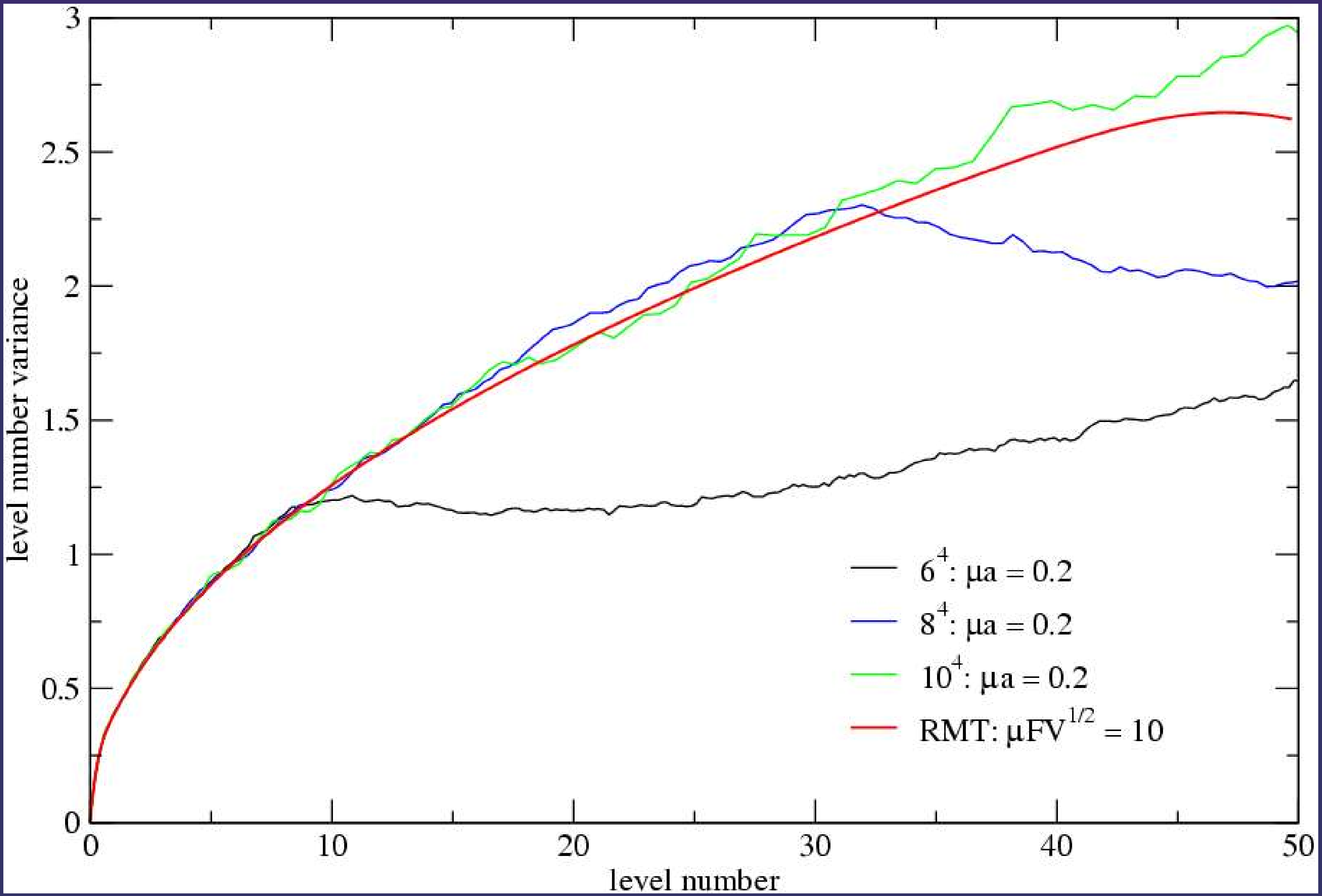
Number Variance (6^4)



Number Variance ($\mu F V^{1/2}$=0.14)



Number Variance ($\mu a=0.2$)



Summary

- Effective theory gives expressions for low energy eigenvalue correlations of $\mu \neq 0$ QCD.
- Fits to lattice eigenvalues agree well and set low energy constants Σ and F.
- Eigenvalue correlations show Thouless energy.
- Thouless energy increases with μ until saturation.