Lattice Study of Transport Coefficients

Extreme QCD

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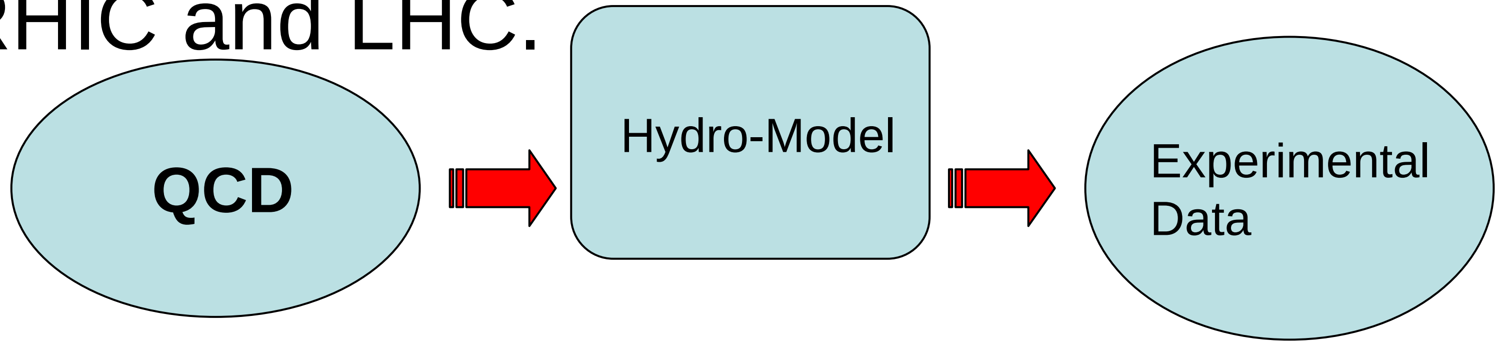
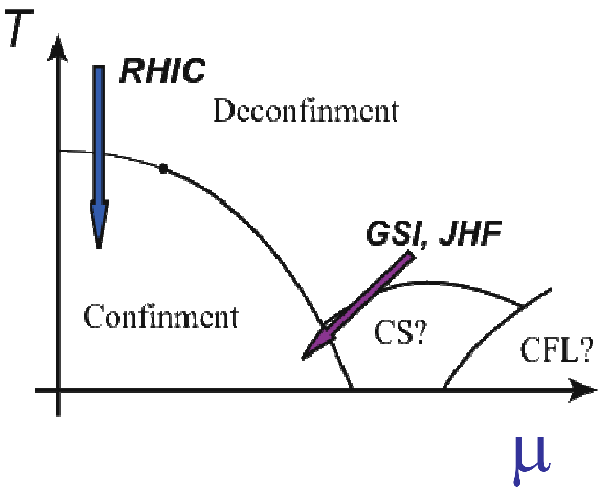
- Introduction
  - New State of Matter at RHIC
- Formulation
- Results
- Summary
- Future Directions ?

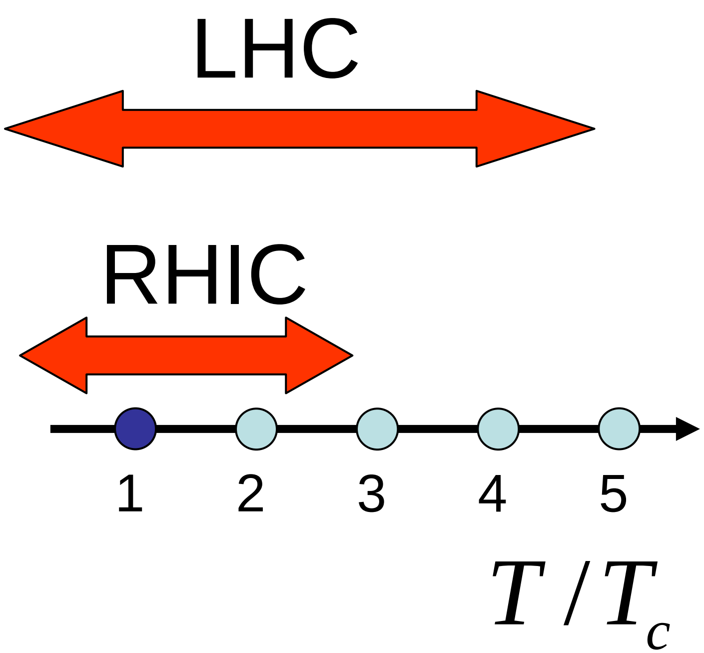
Transport Coefficients

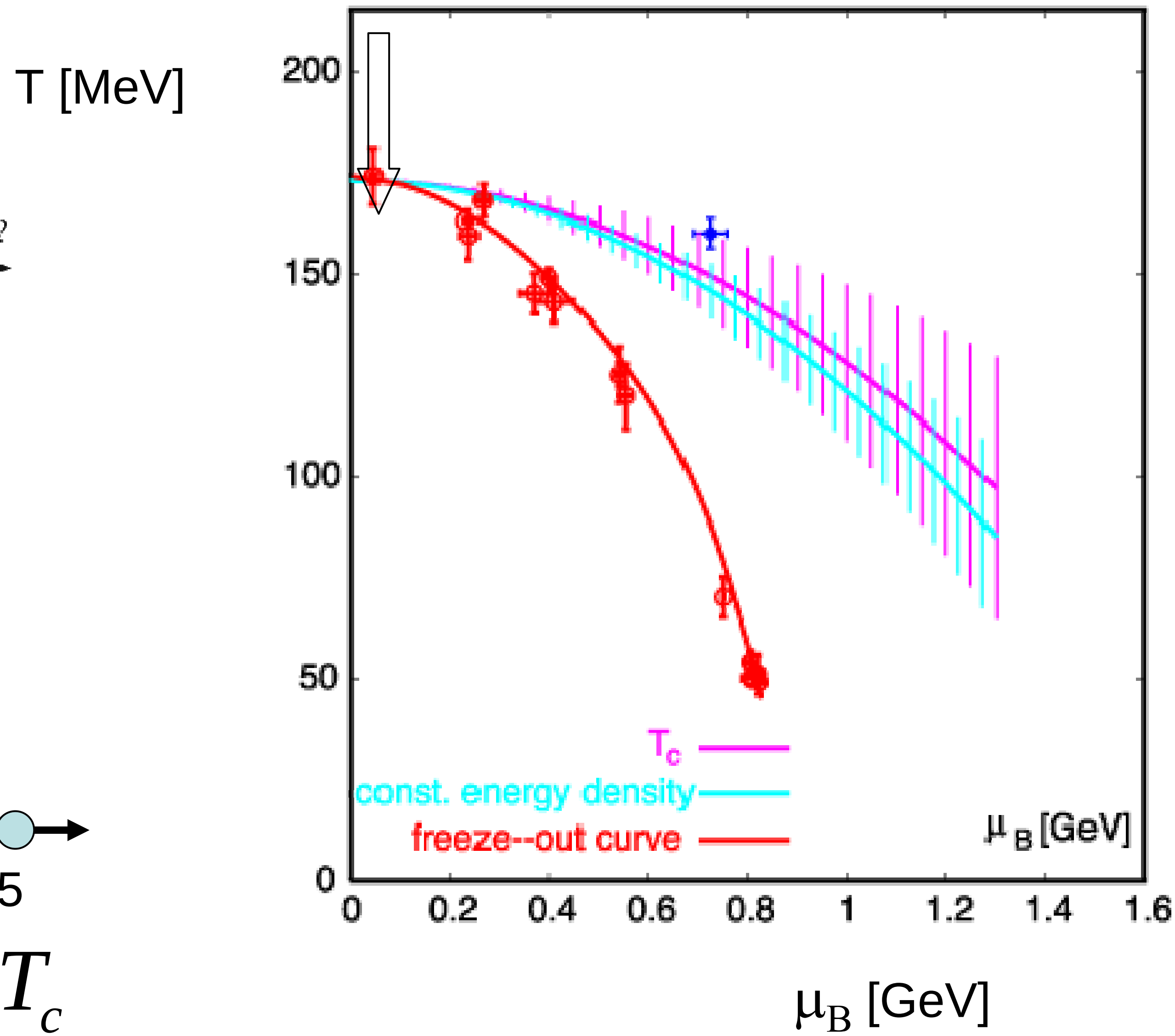
- A Step towards Study of Gluon's Dynamical Behavior – Parameters for non-equilibrium motion.
- They are (in principle) calculable by Lattice
  - using Kubo Formula
  - directly from the spectral function.
- They are important for understanding "a New State of Matter" which is realized in RHIC and LHC.



A Comparison with Lattice Results
P. Braun-Munzinger, K. Redlich and J. Stachel







RHIC-data ⇨ *Big Surprise !*

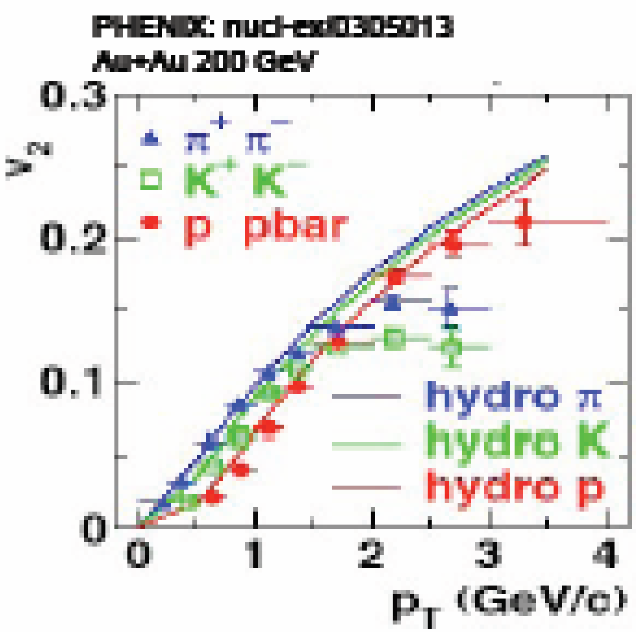
Oh,
really ?

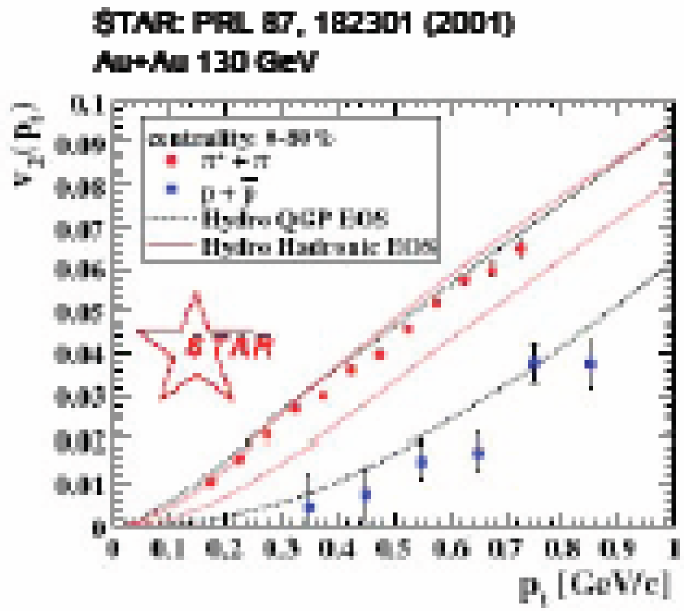
Hydro-dynamical
Model describes
RHIC data well !

At SPS, the Hydro describes well one-particle distributions,

HBT etc., but fails for the elliptic flow.

Hydro describes well v2





Hydrodynamical calculations are based on Ideal Fluid, i.e., zero shear viscosity.

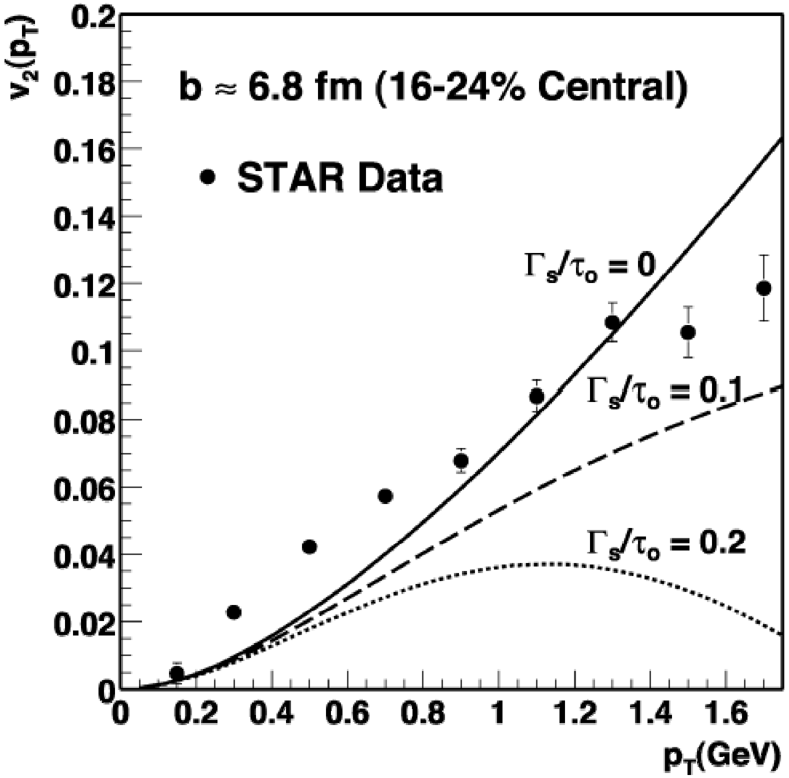
Or not so surprise …

- E. Fermi, Prog. Theor. Phys. 5 (1950) 570
  - Statistical Model
- S.Z.Belen’skji and L.D.Landau, Nuovo.Cimento Suppl. 3 (1956) 15
  - Criticism of Fermi Model
    “Owing to high density of the particles and to strong interaction between them, one cannot really speak of their number.”

Hagedorn, Suppl. Nuovo Cim. 3 (1956) 147. Limiting Temperature

Teaney, nucl-th/0301099

$$\Gamma_s \equiv \frac{\frac{4}{3}\eta}{sT}$$



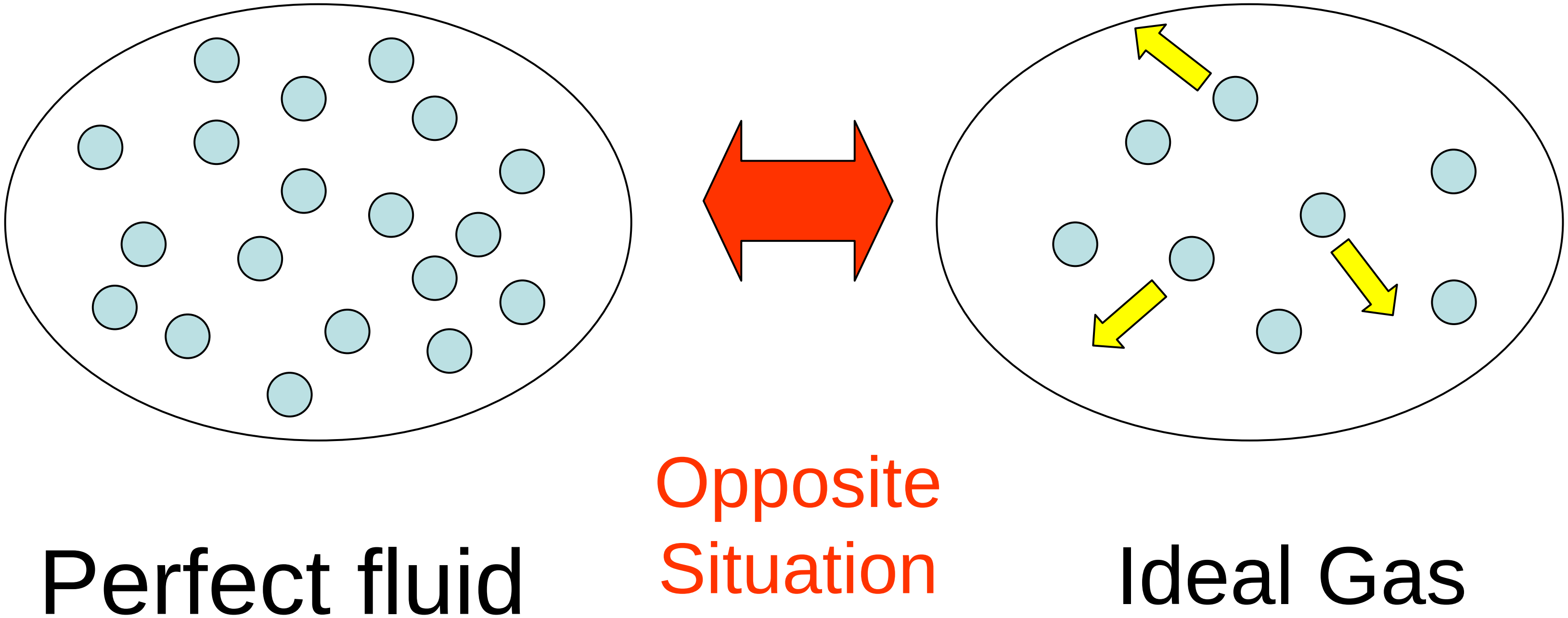
η : shear viscosity

$\tau = \sqrt{t^2 - z^2}$: Time scale of the expansion

Another Big Surprise !

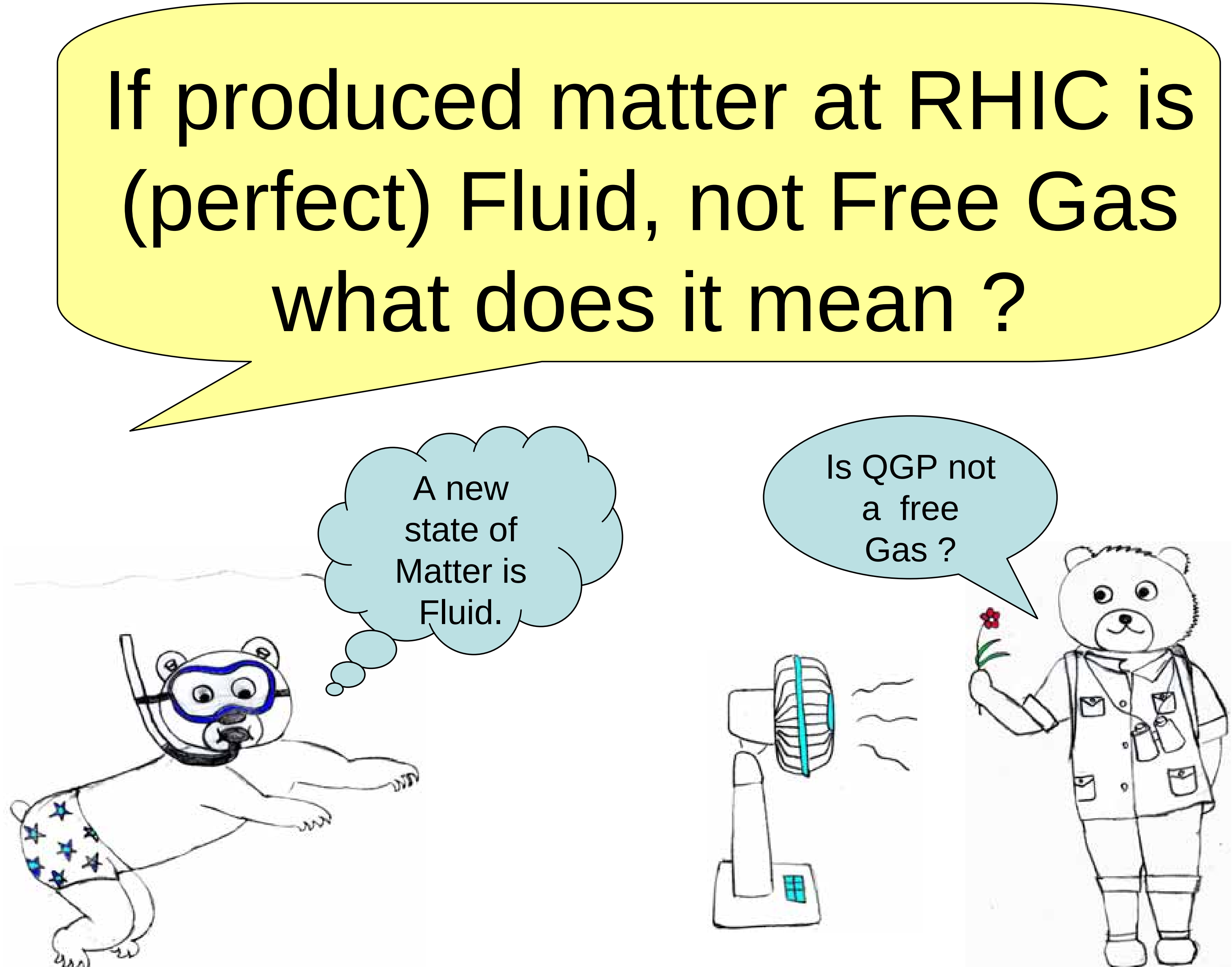
- The Hydrodynamical model assumes zero viscosity,
  i.e.,, Perfect Fluid.
- Phenomenological Analyses suggest also small viscosity.

Ideal Liquid ?



Literature (1)

- Iso, Mori and Namiki, Prog. Theor. Phys. 22 (1959) pp.403-429
  - Applicability Conditions of the Hydrodynamical Model of Multiple Production of Particles from the Point of View of Quantum Field Theory,
    - Correlation Length << System Size
    - Relaxation time << Macroscopic Characteristic Time
    - Transport Coefficients must be small


If produced matter at RHIC is (perfect) Fluid, not Free Gas what does it mean ?
A new state of Matter is Fluid.
Is QGP not a free Gas ?

Lowest Perturbation
(Illustration purpose only)

Pressure

$$P = \frac{\pi^2}{90} T^4 \left(1 - \frac{15}{8} \left(\frac{g}{\pi}\right)^2 + \cdots \right)$$

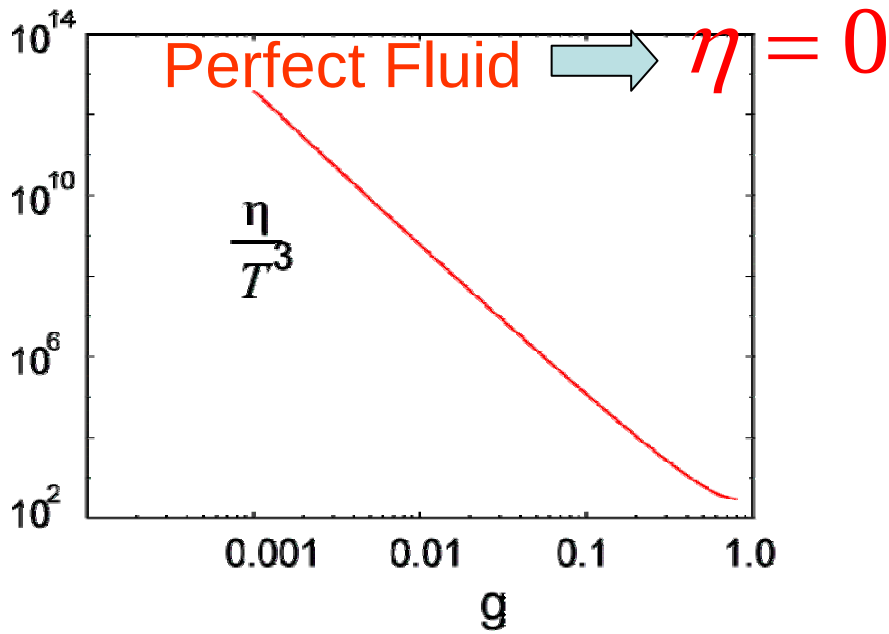
Ideal Free Gas

Viscosity

$$\eta = \kappa \frac{T^3}{g^4 \ln g^{-1}}$$

$$\kappa = 27.126 (N_f = 0), \quad 86.473 (N_f = 2)$$

- At weak coupling, it increases.



Literature (2)

- Hosoya, Sakagami and Takao, Ann. Phys. 154 (1984) 228.
- Hosoya and Kayantie, Nucl. Phys. B250 (1985) 666.
- Horsley and Shoenmaker, Phys. Rev. Lett. 57 (1986) 2894; Nucl. Phys. B280 (1987) 716.
- Karsch and Wyld, Phys. Rev. D35 (1987) 2518.
- Aarts and Martinez-Resco, JHEP0204 (2002)053, hep-lat/0209033(Lattice02), hep-ph/0203177.

Literature (3)

- Masuda, A.N.,Sakai and Shoji
  Nucl.Phys. B(Proc.Suppl.)42, (1995),526
- A.N., Sakai and Amemiya
  Nucl.Phys. B(Proc.Suppl.)53, (1997), 432
- A.N, Saito and Sakai
  Nucl.Phys. B(Proc.Suppl.)63, (1998), 424
- Sakai, A.N. and Saito
  Nucl.Phys. A638, (1998), 535c
- A.N, Sakai
  hep-lat/0406009

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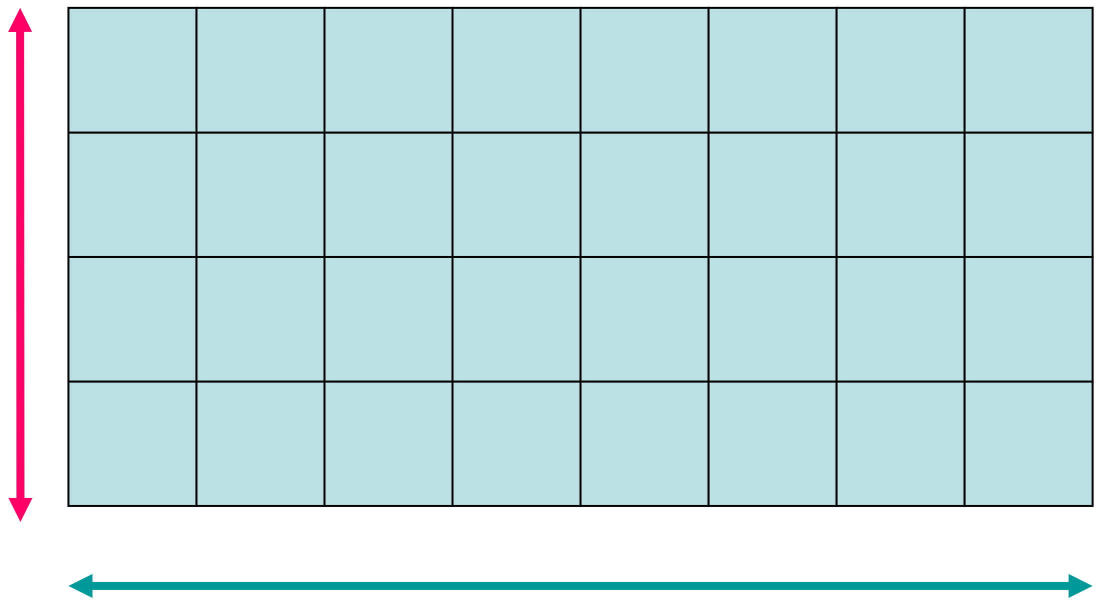
Some Special Features of Lattice QCD at Finite Temperature and Density

$$N_t a_t = \frac{1}{kT}$$



$$N_s a_s$$

High Temperature → $N_t a_t$: small

Energy Momentum Tensors

$$T_{\mu\nu} = 2Tr(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})$$

$$(T_{\mu\mu} = 0)$$

$$U_{\mu\nu}(x) = \exp(ia^2 gF_{\mu\nu}(x))$$

$$F_{\mu\nu} = \log U_{\mu\nu} / ia^2 g$$

or

$$F_{\mu\nu} = (U_{\mu\nu} - 1) / ia^2 g$$

Real Time Green function vs. *Temperature* Green function

Hashimoto, A.N. and Stamatescu, Nucl.Phys.B400(1993)267

$$<<\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]>>\equiv\frac{1}{Z}\mathrm{Tr}(\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]e^{-\beta H})$$

$$=F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Lambda(\omega,\vec{p})$$

$$\phi(t,\vec{x})=e^{itH}\phi(0,\vec{x})e^{-itH}$$

$$G_{\beta}^{ret/adv}(t,\vec{x};t',\vec{x}')=\pm\theta(t-t'/t'-t)<<...>>$$

$$=F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}K_{\beta}^{ret/adv}(\omega,\vec{p})$$

$$K_{\beta}^{ret/adv}(\omega,\vec{p})=\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Lambda(\omega')}{\omega-\omega'\pm\varepsilon}$$

Temperature Green function

$$G_\beta(\tau,\vec{x};\tau',\vec{x}') = << T_\tau \phi(\tau,\vec{x})\phi(\tau',\vec{x}') >>$$

$$\phi(t,\vec{x}) = e^{\tau H}\phi(0,\vec{x})e^{-\tau H}$$

$$G_\beta(\tau,\vec{x};0,0) = G_\beta(\tau+\beta,\vec{x};0,0)$$

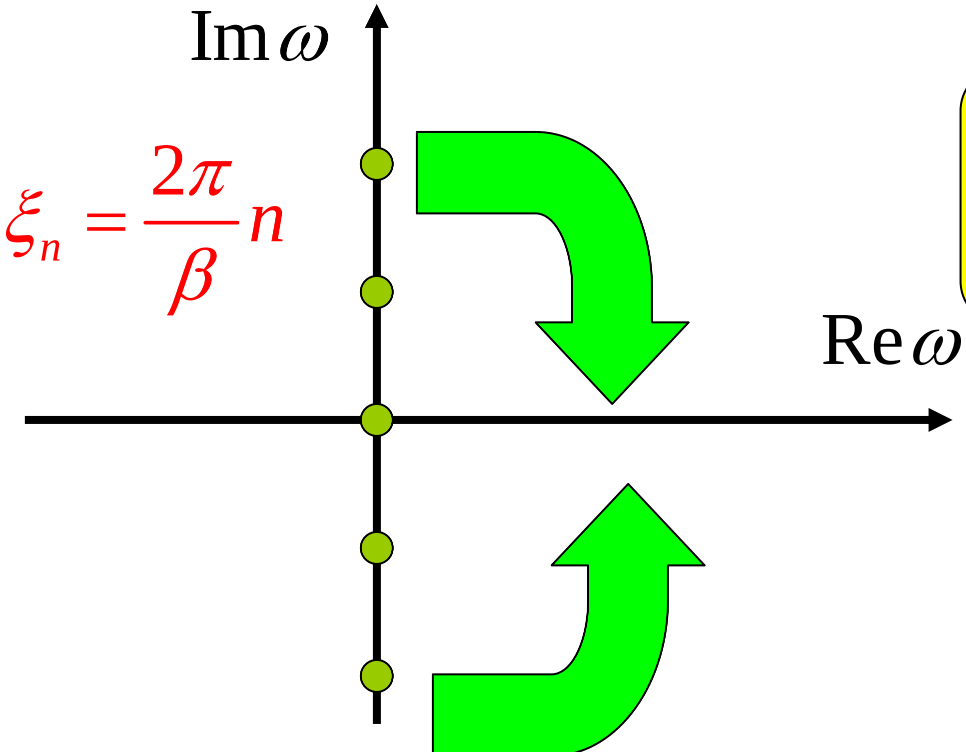
$$\hat{K}_\beta(\xi_n,\vec{p}) = F^{-1}\int_0^\beta d\tau e^{-i\xi_n(\tau-\tau')}G_\beta(\tau,\vec{x};\tau',\vec{x}')$$

$$\xi_n = \frac{2\pi}{\beta}n, n = 0,\pm 1,\pm 2,,,$$

Matsubara-frequencies

Abrikosov-Gorkov-Dzyalosinski-Fradkin Theorem

$$\hat{K}_\beta(\xi_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Lambda(\omega)}{\omega - i\xi_n} = iK_\beta(i\xi_n)$$



On the lattice, we measure Temperature Green function at $\omega = \xi_n$

We must reconstruct Advance or Retarded Green function.

Kubo’s Linear Response Theory

- Zubarev
  “Non-Equilibrium Statistical Thermo-dynamics”
- Kubo, Toda and Saito
  “Statistical Mechanics”

Transport Coefficients of QGP

We measure Correlations of Energy-Momentum tensors $< T_{\mu\nu}(0) T_{\mu\nu}(\tau) >$

Convert them (Matsubara Green Functions) to Retarded ones (real time).

Transport Coefficients (Shear Viscosity, Bulk Viscosity and Heat Conductivity)

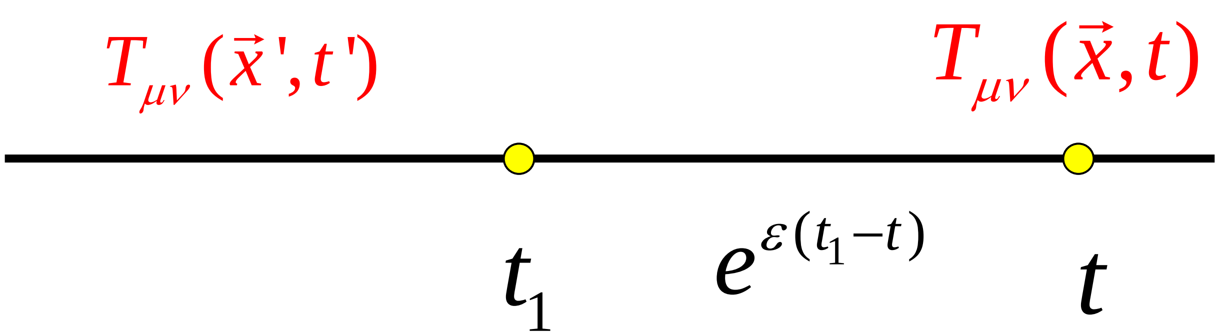
$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{12}(\vec{x}, t) T_{12}(\vec{x}', t') >_{ret}$$

$$\frac{4}{3}\eta + \varsigma = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{11}(\vec{x}, t) T_{11}(\vec{x}', t') >$$

$$\chi = -\frac{1}{T} \int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{01}(\vec{x}, t) T_{01}(\vec{x}', t') >_{ret}$$

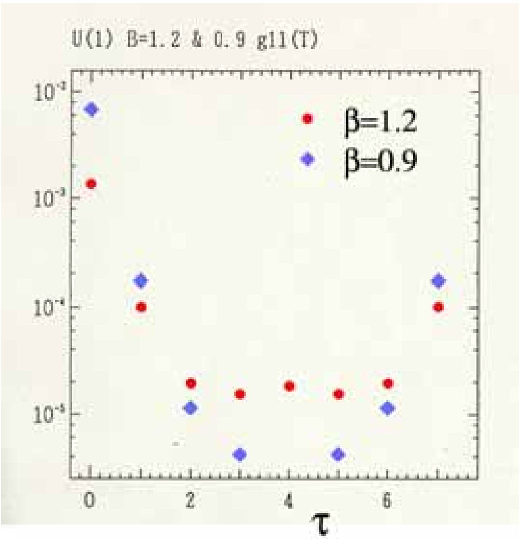
η **: Shear Viscosity** ς **: Bulk Viscosity**

χ **: Heat Conductivity** ⇨ we do not consider in Quench simulations.



$$-\infty < t' < t_1 < t$$

Correlators



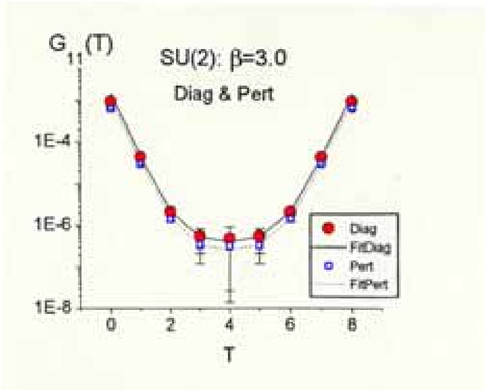
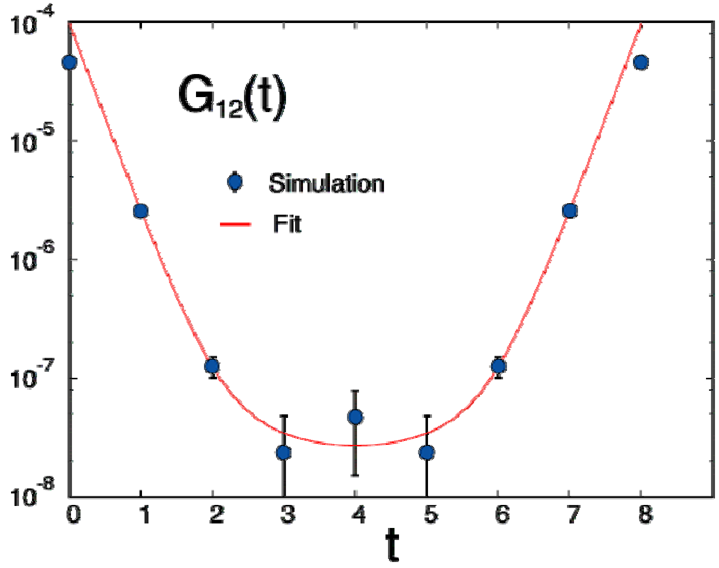


Figure 3. Fit of $G_{11}(T)$ by the parameters of spectral function for SU(2) at $\beta = 3.0$



U(1)
Coulomb and Confinement Phases

SU(2)

Two Definitions:

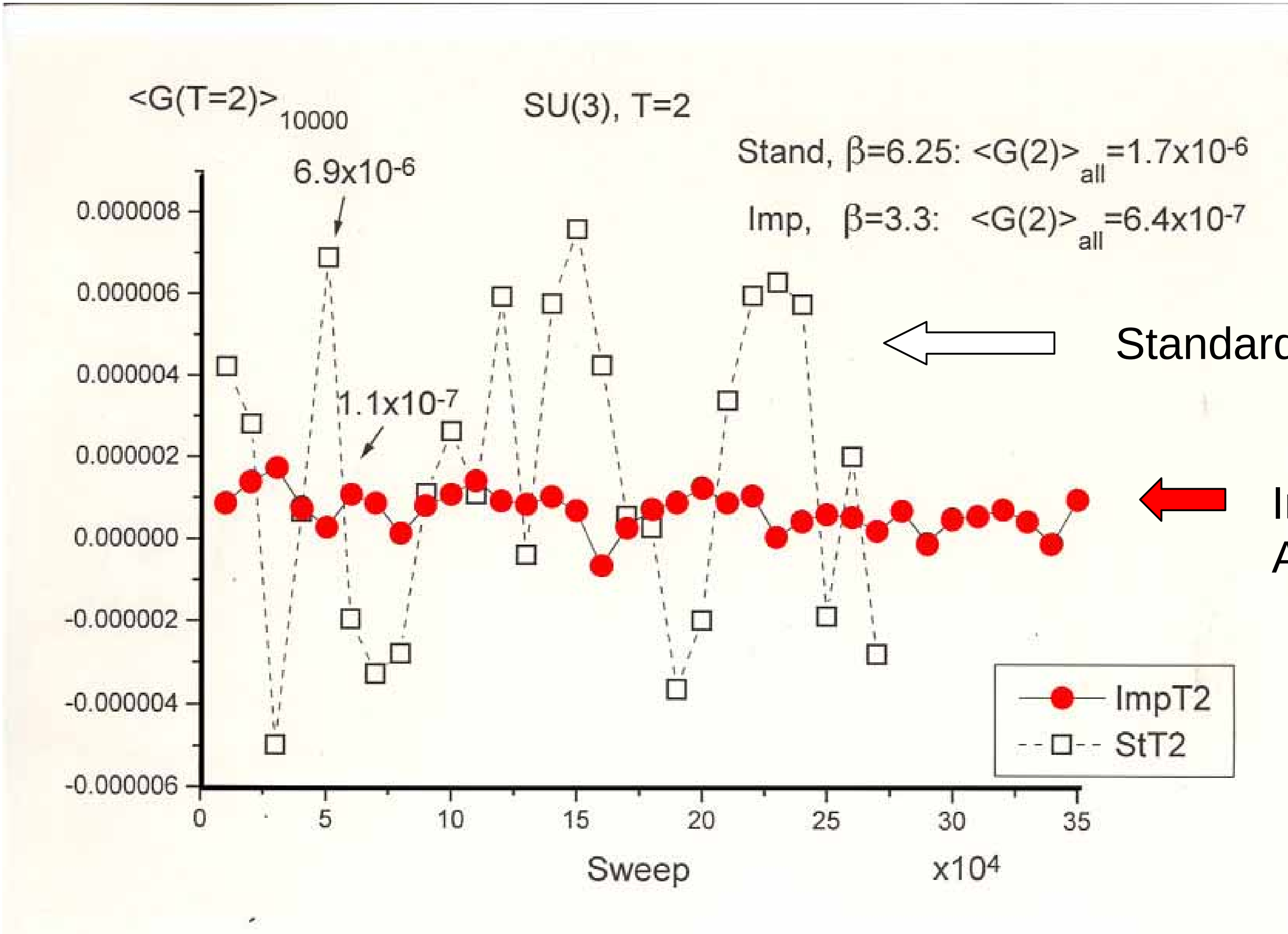
F=log U

F=U-1

SU(3)

Improved Action

Fluctuations in MC sweeps



Standard Action

Improved Action

Errors in U(1), SU(2), SU(3) standard and SU(3) improved

1995 U(1)

1997 SU(2)

1998 SU(3) preliminary

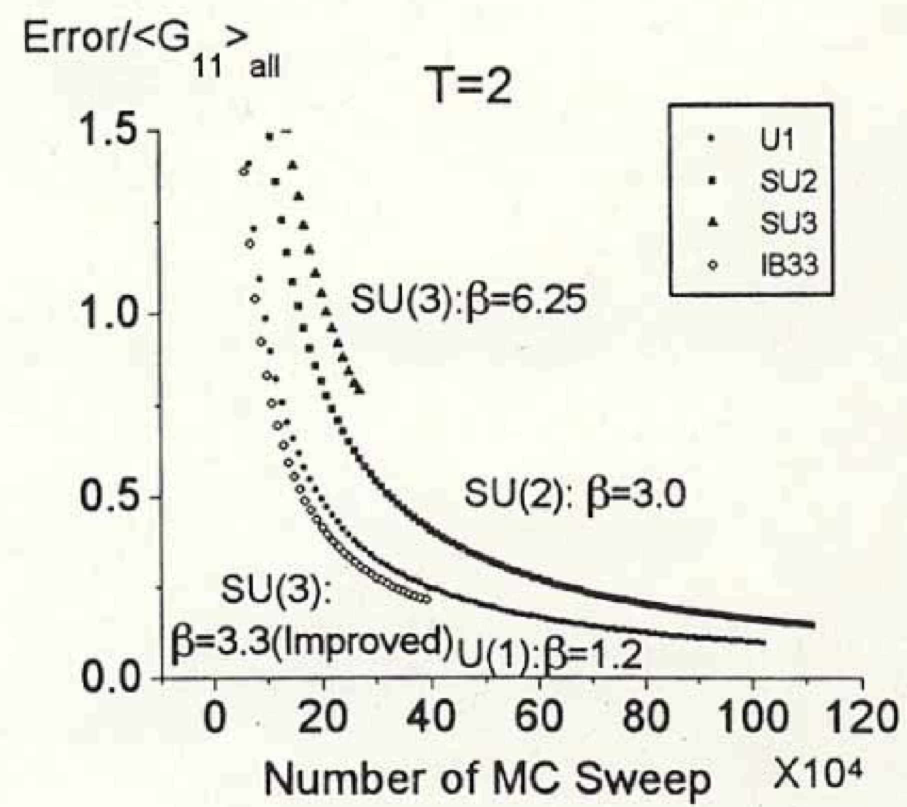


Figure 2. Error as a function of number of MC sweeps at $T = 2$ for U(1) $\beta = 1.2$, SU(2) $\beta = 3.0$, SU(3) $\beta = 6.25$ and improved action for SU(3) $\beta = 3.3$.

Important Player is the Spectral Function $\rho(\omega)$

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

Horsley and Shoenmaker

Numerically it is hard to determine $\rho(\omega)$

from $< T_{\mu\nu}(0) T_{\mu\nu}(\tau) >$ without ambiguity.

Assumption for the Spectral Functions

We measure Matsubara Green Function on Lattice (in coordinate space).

$$< T_{\mu\upsilon}(t,\vec{x}) T_{\mu\upsilon}(0) > = G_{\beta}(t,\vec{x}) = F.T.G_{\beta}(\omega_n, \vec{p})$$

$$G_{\beta}(\vec{p}, i\omega_n) = \int d\omega \frac{\rho(\vec{p},\omega)}{i\omega_n - \omega}$$

We assume (Karsch-Wyld)

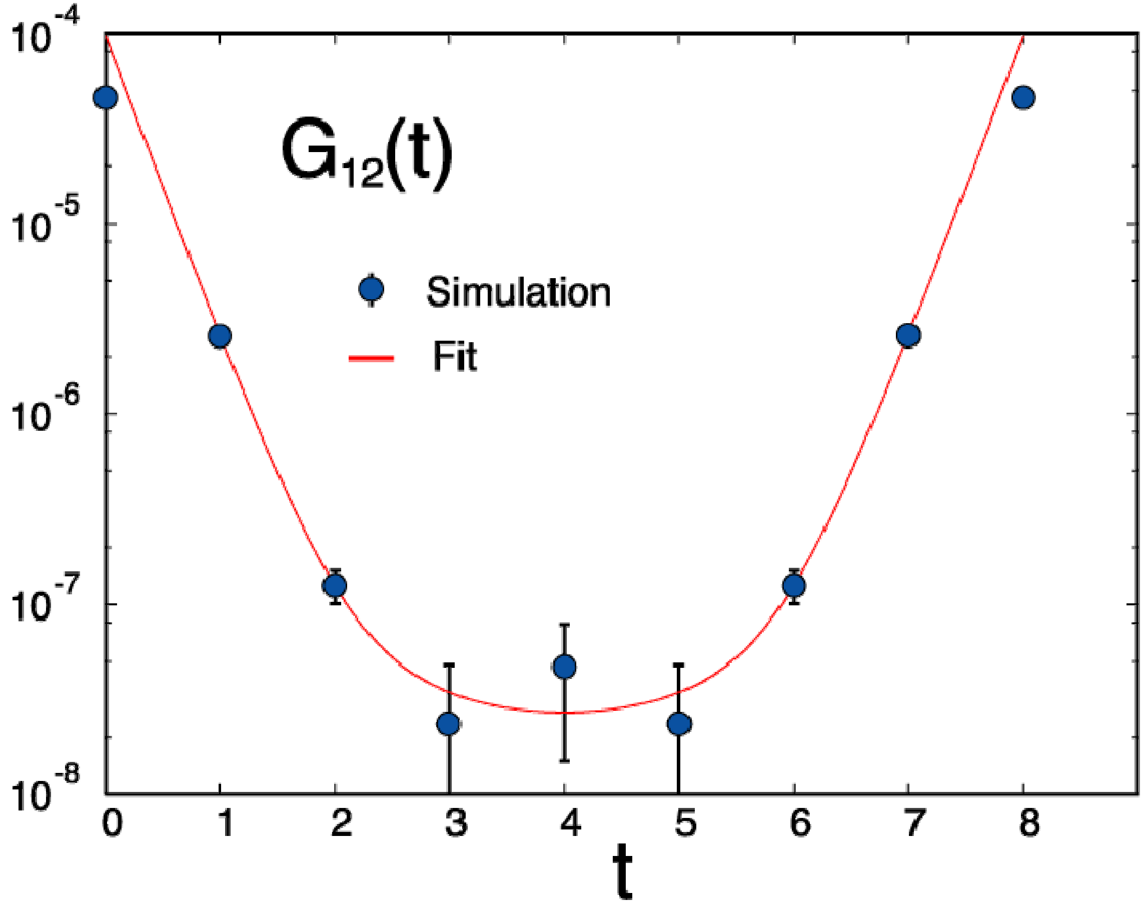
$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2 + \gamma^2} + \frac{\gamma}{(m+\omega)^2 + \gamma^2} \right)$$

and determine three parameters,

A, m, γ.

We need large Nt !

Nt=8



Lattice and Statistics

Iwasaki Improved Action

$$16^3 \times 8$$

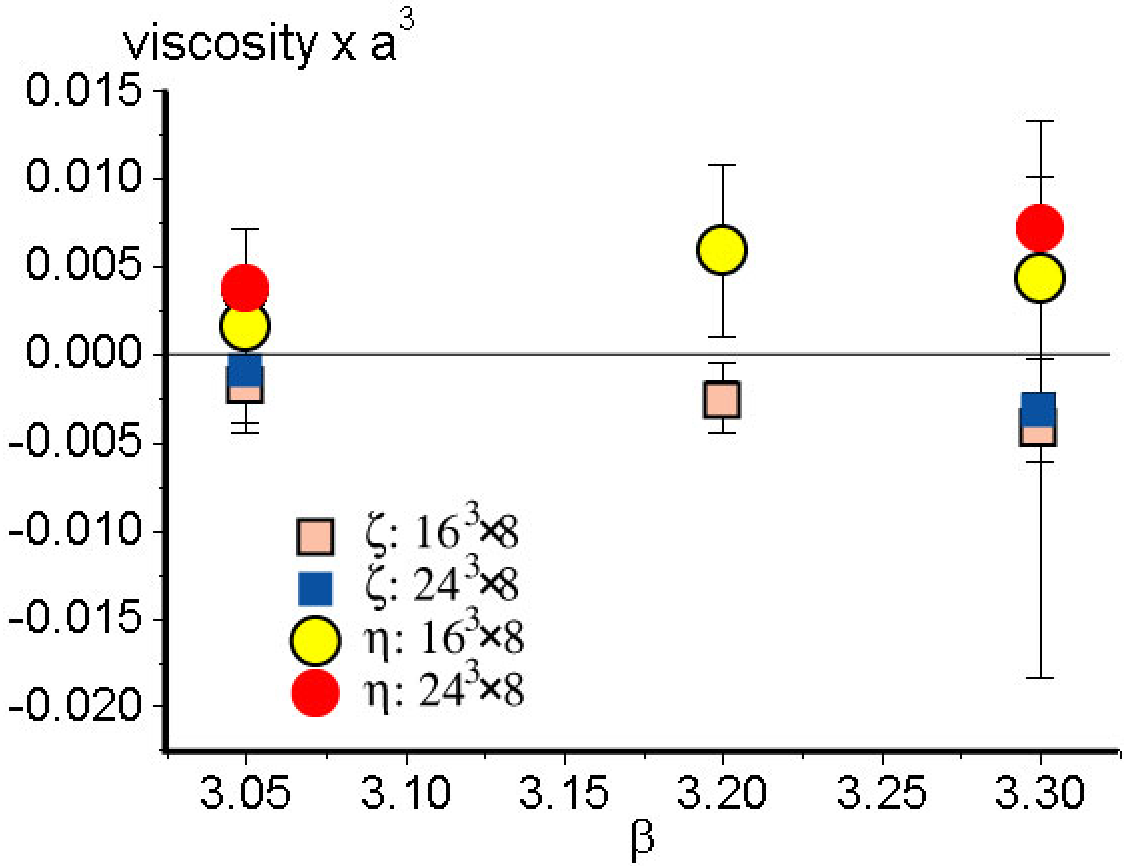
$\beta=3.05$: 1333900 sweeps
$\beta=3.20$: 1212400 sweeps
$\beta=3.30$: 1265500 sweeps

$$24^3 \times 8$$

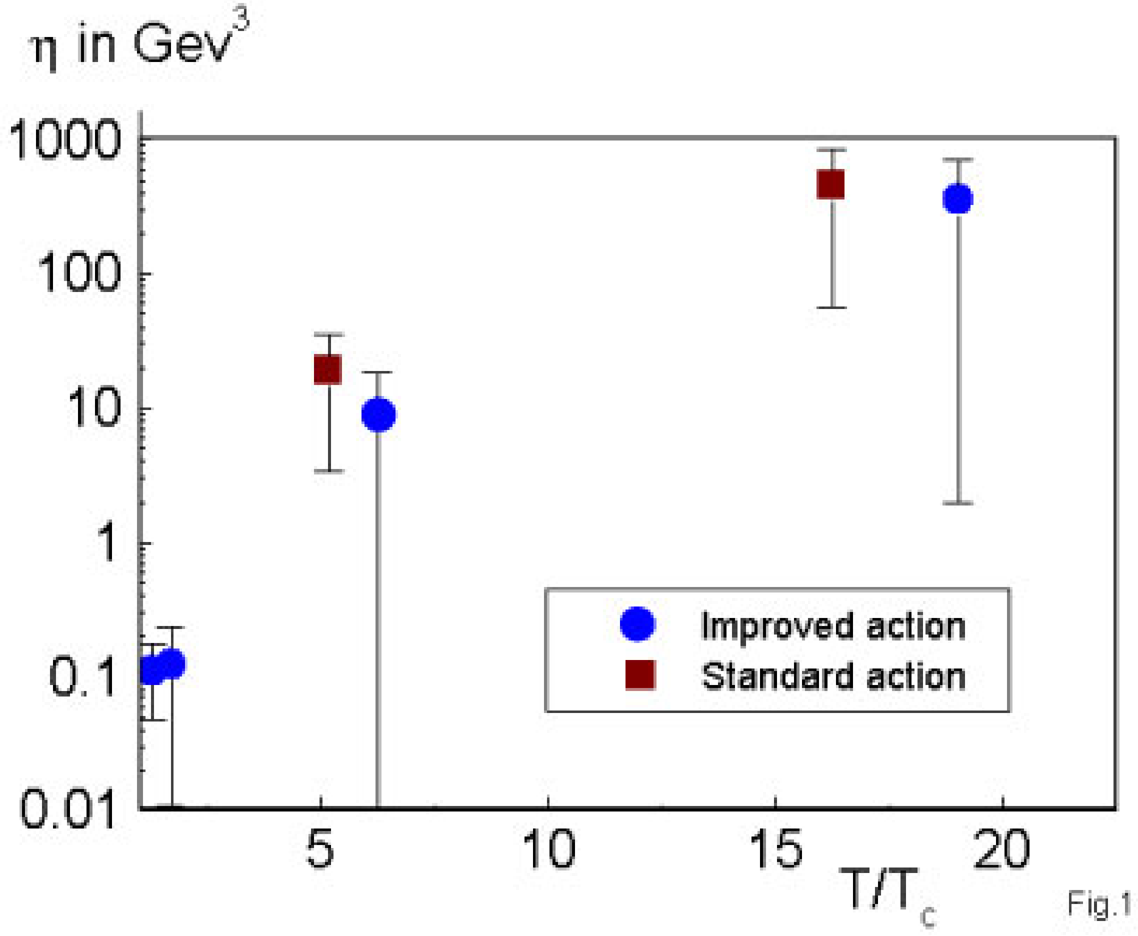
$\beta=3.05$: 61000 sweeps
$\beta=3.30$: 84000 sweeps

Quench

Results: Shear and Bulk Viscosities

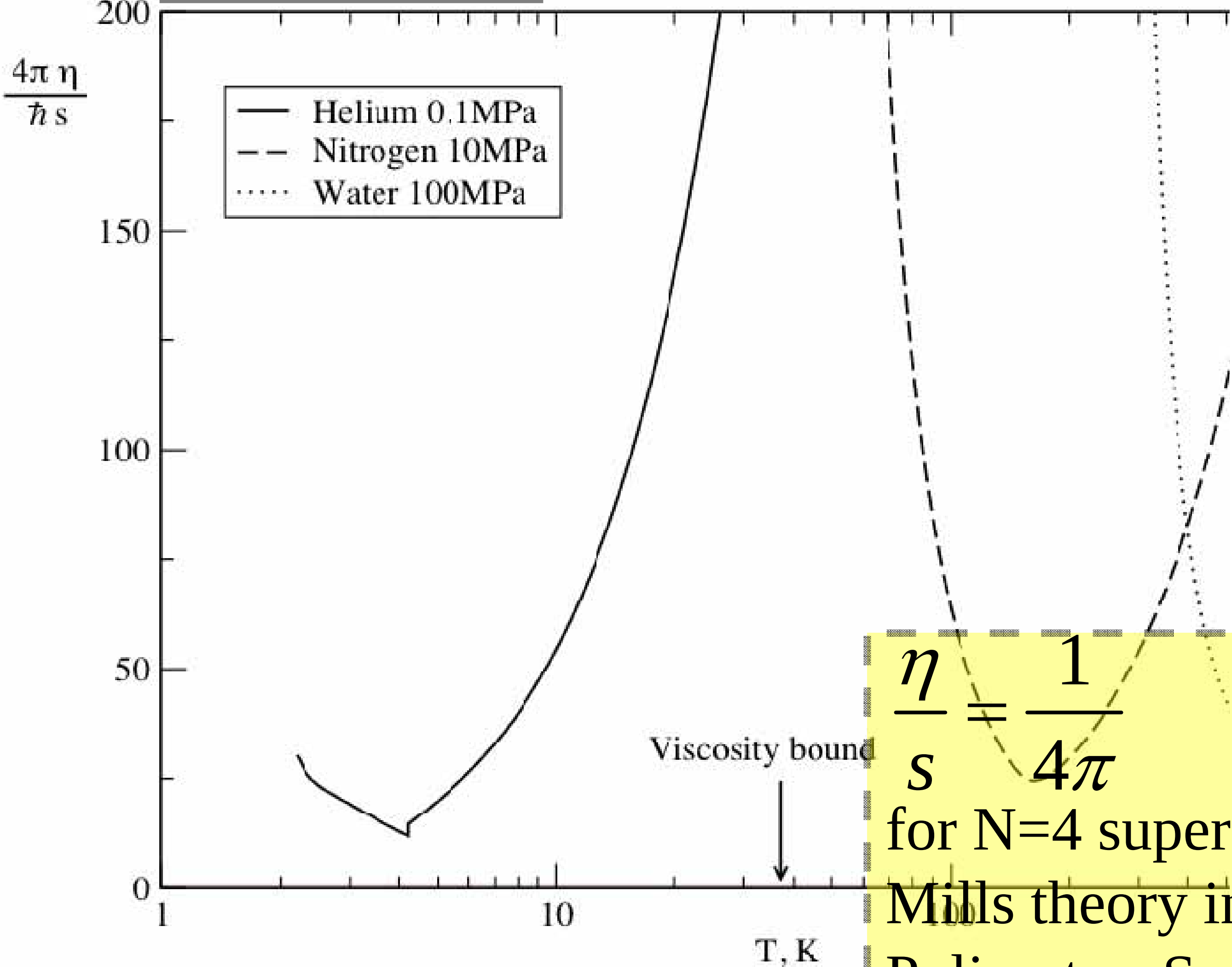


Very high Temperature



$$\frac{\eta}{s} \geq \frac{1}{4\pi} \; !$$

Kovtun, Son and Starinets, hep-th/0405231



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

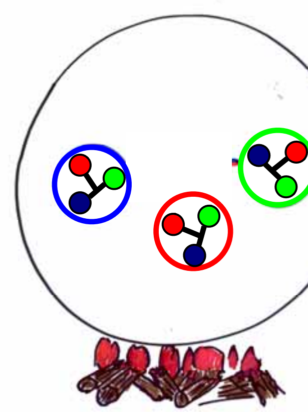
for N=4 supersymmetric Yang-Mills theory in the large N.

Policastro, Son and Starinets, Phys Rev. Lett. 87 (2001) 081601

$\frac{\eta}{s}$ can have the lower limit ?

- Counter Example by Prof. Baym
  - We heat up Billiard Balls which have inter-structure. Then Entropy increases. If the surface of the balls does not change, the Viscosity should be the same.

$$\Rightarrow \quad \frac{\eta}{s} \to 0$$



- We may give Counter-Argument

Entropy Density

$$F = fV$$

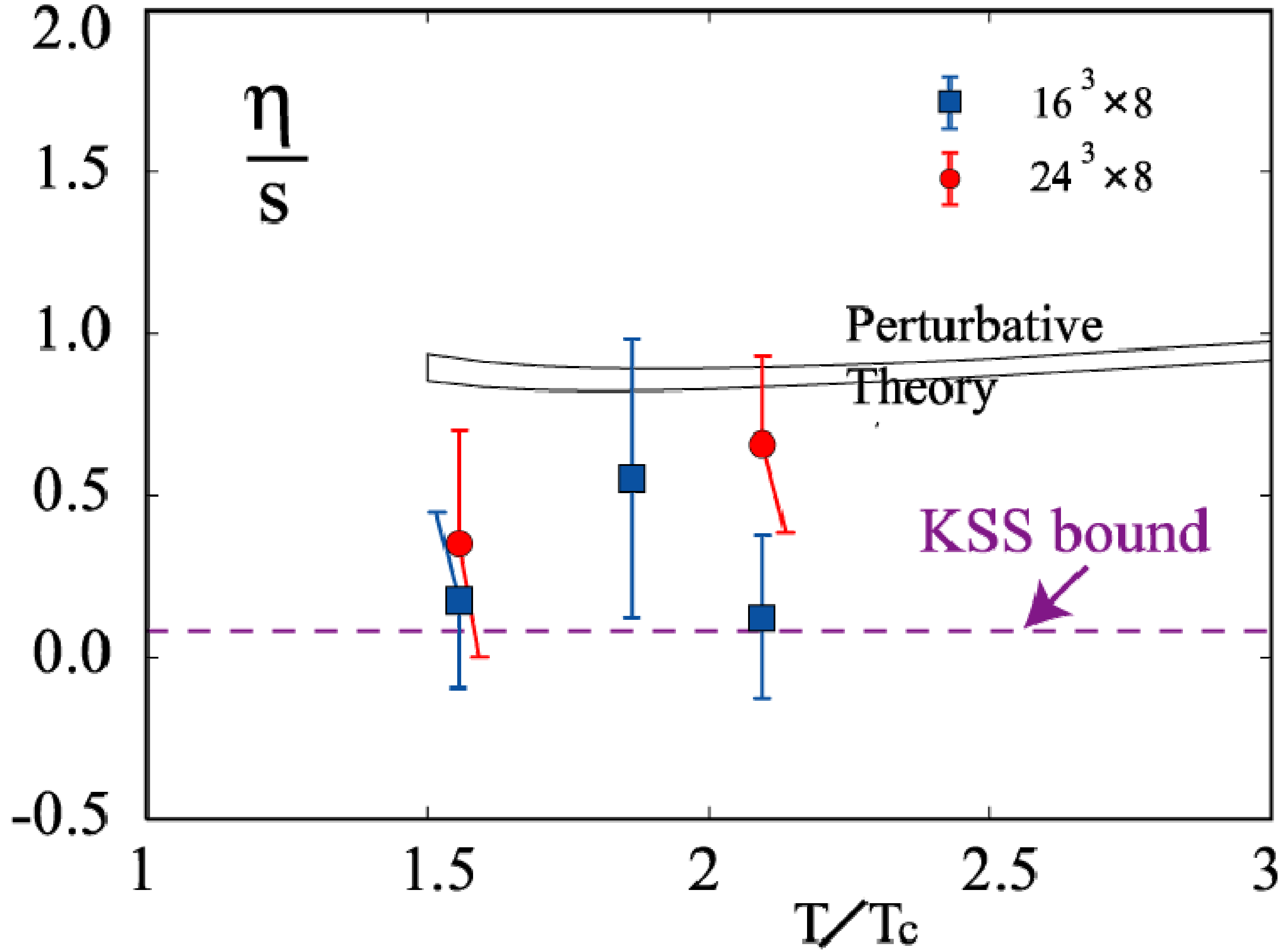
$$f = -p$$

$$U - TS = -T \log Z = F$$

$$\Rightarrow \quad s = \frac{S}{V} = \frac{\varepsilon + p}{T}$$

$$\left.\frac{p}{T^4}\right|_{\beta_0}^{\beta} = \int_{\beta_0}^{\beta} d\beta' N_T{}^4 \left(\langle S \rangle_T - \langle S \rangle_0 \right)$$

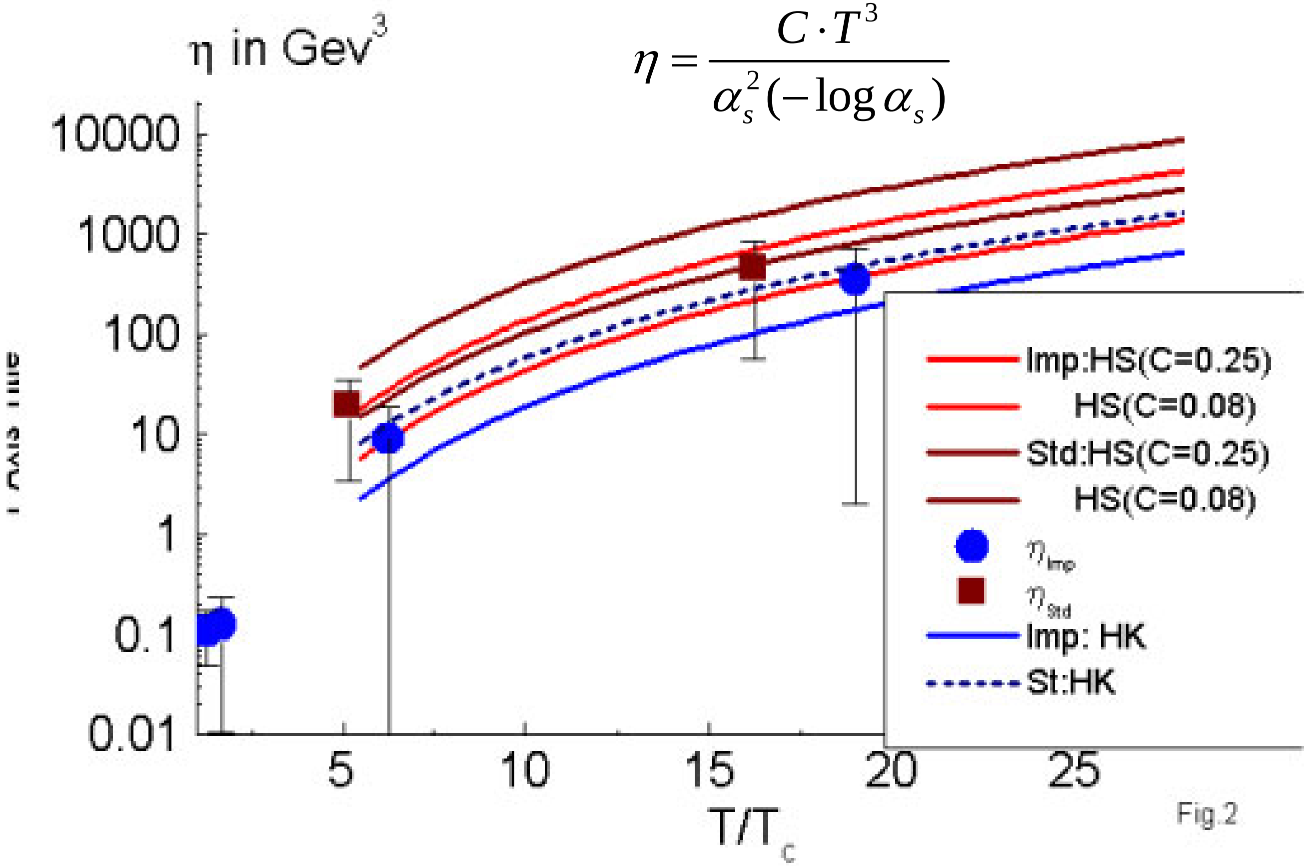
We reconstruct p from <S> by CP-PACS
(Okamoto et al., Phys.Rev.D (1999) 094510)



2.0
1.5
1.0
0.5
0.0
-0.5
η/s
16³×8
24³×8
Perturbative Theory
KSS bound
1
1.5
2
2.5
3
T/Tc

Comparison with Pertubative Calculations

$$\eta = \frac{C \cdot T^3}{\alpha_s^2(-\log \alpha_s)}$$



Good for T/Tc>5

Spectral Function by Aarts and Resco

$$\rho(\omega) = \rho^{low}(\omega) + \rho^{high}(\omega)$$

$$\frac{\rho^{low}(\omega)}{T^4} = x \frac{b_1 + b_2 x^2 + \cdots}{1 + c_1 x^2 + c_2 x^4 + \cdots} \qquad x \equiv \frac{\omega}{T}$$

$$\rho^{high}(\omega) = \theta(\omega - 4m^2) \frac{(\omega^2 - 4m^2)^{5/2}}{48\pi\omega} [n(\omega) + 0.5]$$

Fitting with three parameters, $b_1 \quad c_1 \quad m$

$\Rightarrow$ $c_1 < 0$?

Summary

- We have calculated Transport Coefficients on Nt=8 Lattice:
  - Quench Approximation
  - We can fit three parameters in the Spectral Function:

$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2}+\frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

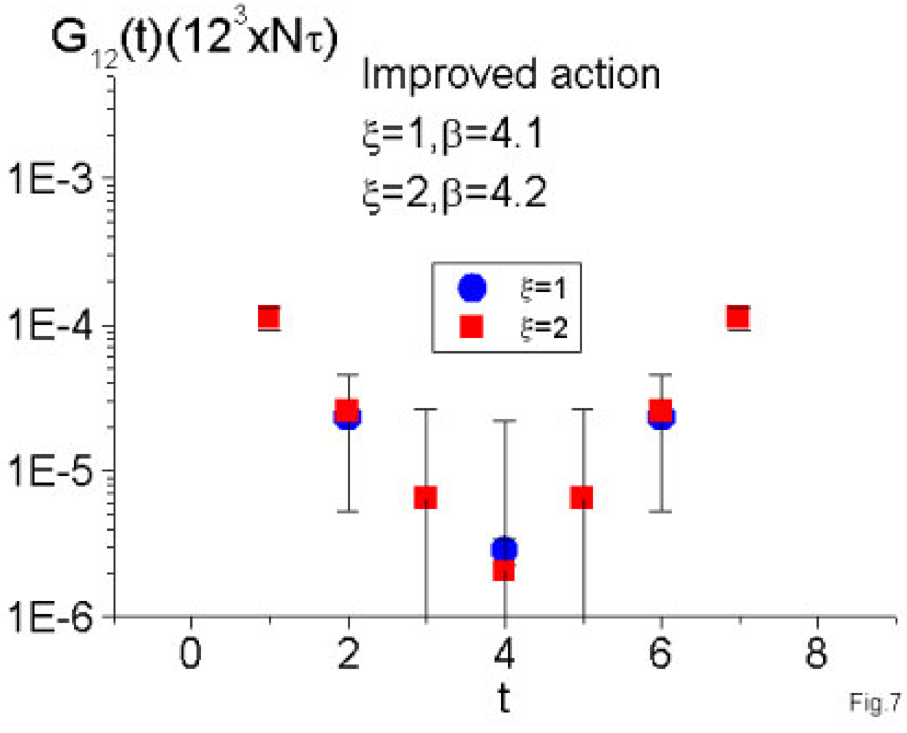
- Shear Viscosity
  - Positive $\eta / s \sim 0.1$
- Bulk Viscosity ~ 0
- Improved Action works well to get good Signal/Noise ratio. $T_{\mu\nu}$

Future direction ?

- More Points at different Temperature
  - Now on-going …
- Does Results change if we employ other Assumption ?
  - We are trying …
- Full QCD ?

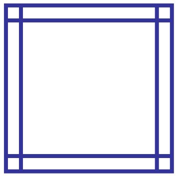
Future direction ? (continued)

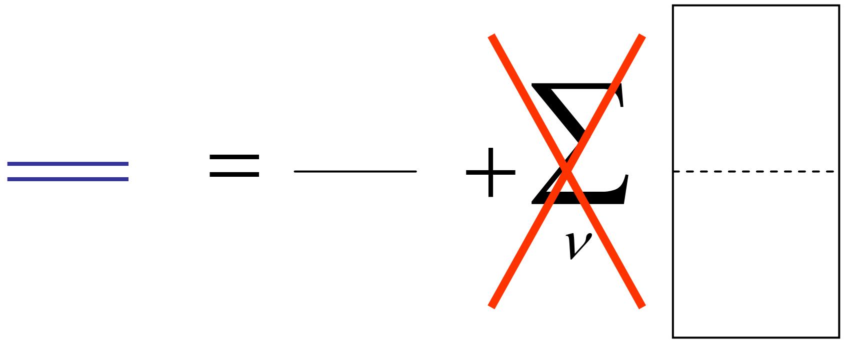
- Anisotropic lattice has matured and will help us to get more data points to determine the spectral function.



Future Direction ? (continued)

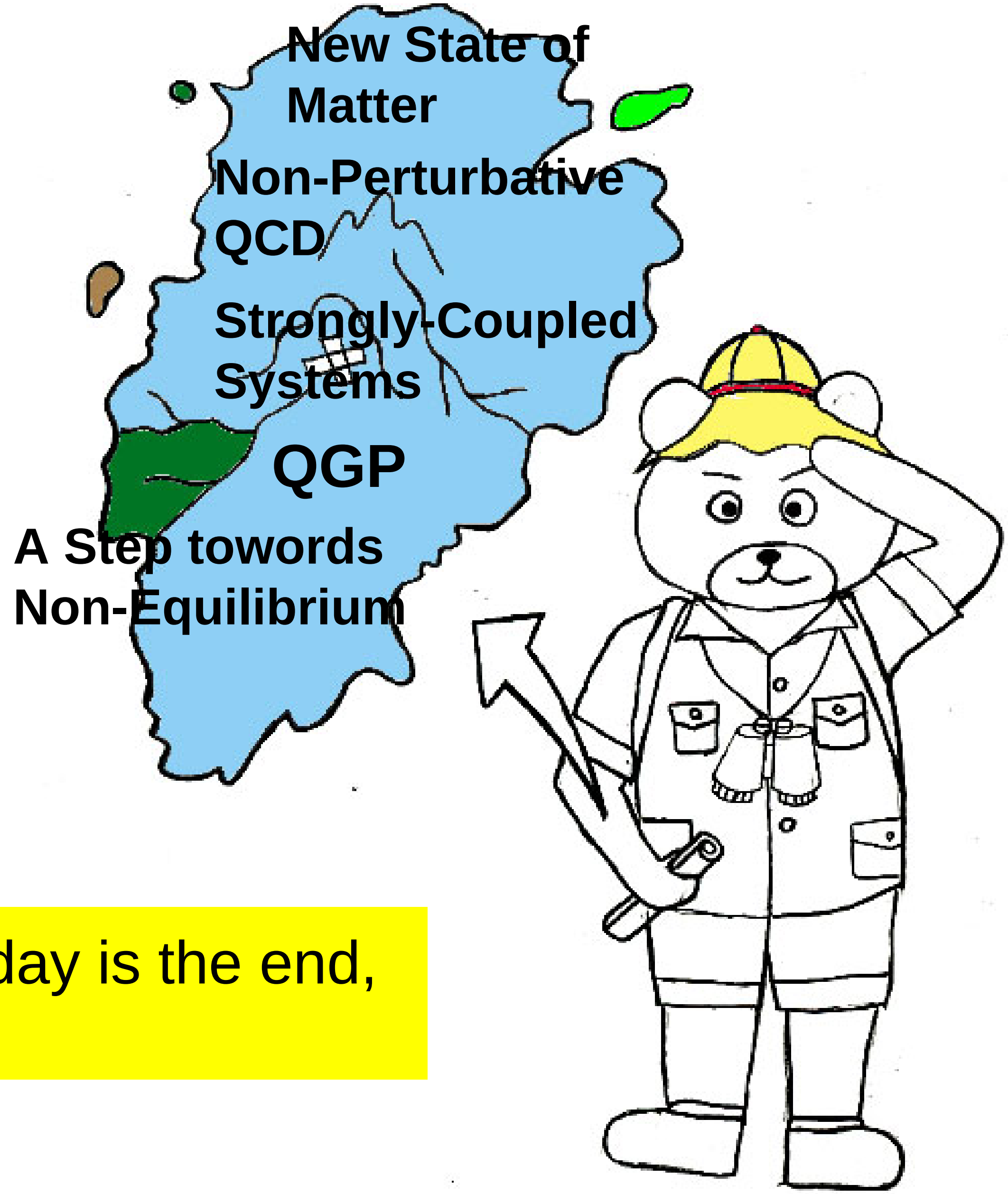
- $F_{\mu\nu}$ with Fat-Links ?

$F_{\mu\nu} =$ Any Closed Loops in μ−ν plane $=$ 



Anyway

- Let us Study Transport Coefficients by Lattice !
- And Contribute to Study New State of Matter from Lattice QCD.



- What we showed today is the end, but a starting point.