# Flow equations for strong QCD

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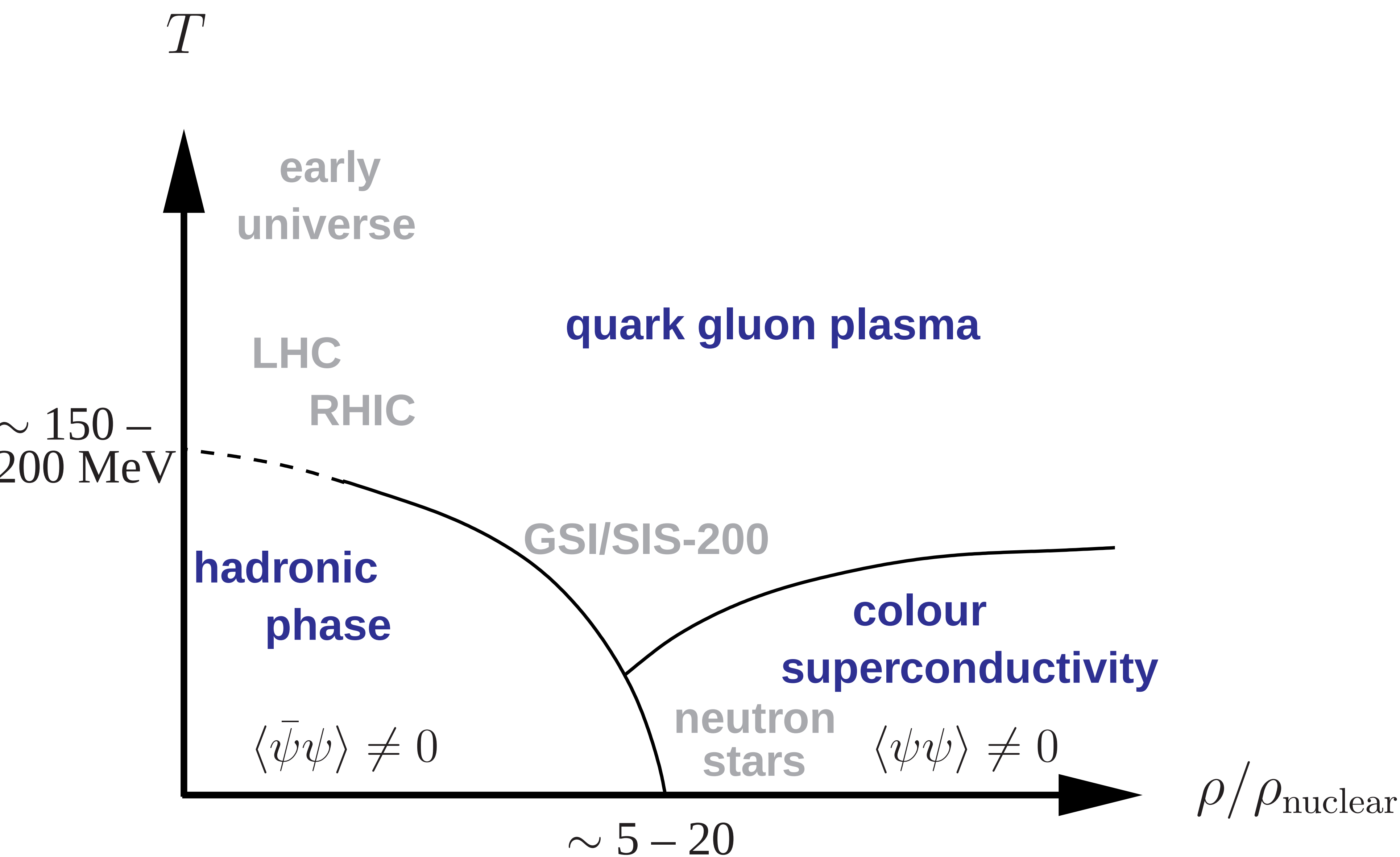
**supported by:** EPSRC Advanced Research Fellowship

**based on collaborations with:** S. Nedelko, J.M. Pawlowski, L. v. Smekal

# motivation


$T$
early
universe
LHC
RHIC
quark gluon plasma
$\sim 150$ –
200 MeV
GSI/SIS-200
hadronic
phase
colour
superconductivity
neutron
stars
$\langle \bar{\psi}\psi \rangle \neq 0$
$\langle \psi\psi \rangle \neq 0$
$\rho/\rho_{\rm nuclear}$
$\sim 5 - 20$

# motivation

- **physics of ’strong QCD’**

  **confinement**
  **confinement-deconfinement phase transition**

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- **methods**

  discretisation: lattice

  functional methods:

    Schwinger-Dyson eqs
    stochastic quantisation
    renormalisation group methods

# renormalisation group

- **flow equation**
  successive integrating-out of momentum modes via momentum cutoff $S \longrightarrow S + \Delta S_k$, with

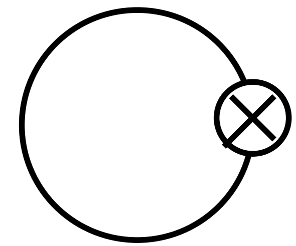
$$\Delta S_k = \int_q \phi(q)\, R_k(q)\, \phi(-q)$$

# renormalisation group

- **flow equation**

  **leads to flow for effective action $\Gamma_k$ ($t = \ln k$)**

$$\partial_t \Gamma_k = \frac{1}{2}\, \mathrm{Tr} \left[ \left( \frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\, \delta\phi} + R_k \right)^{-1} \partial_t R_k \right] = \frac{1}{2}$$



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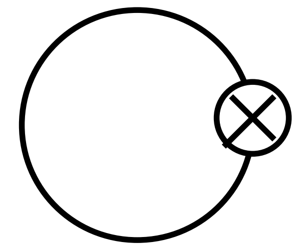

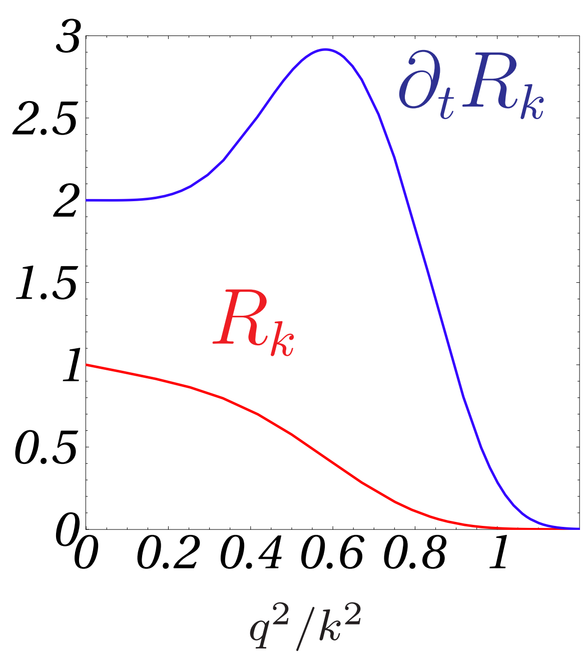

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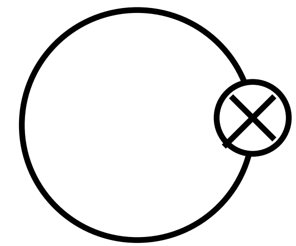

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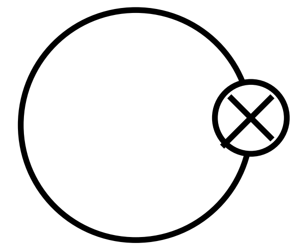

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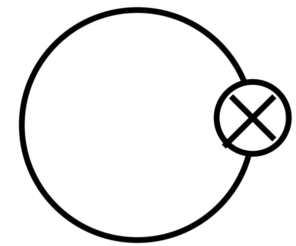

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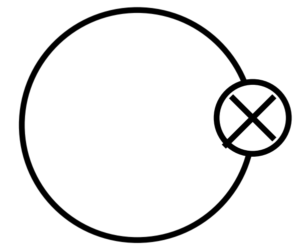

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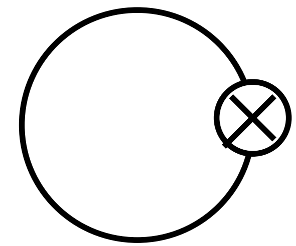

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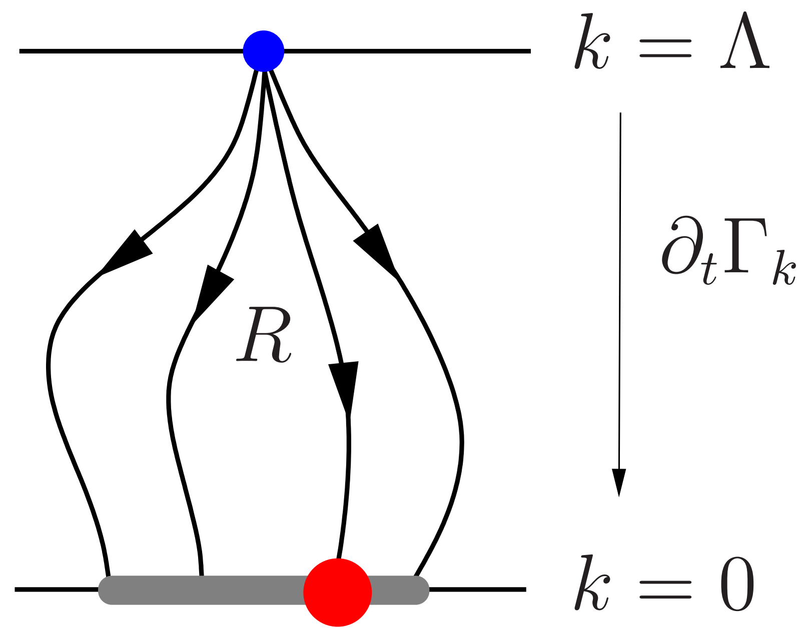

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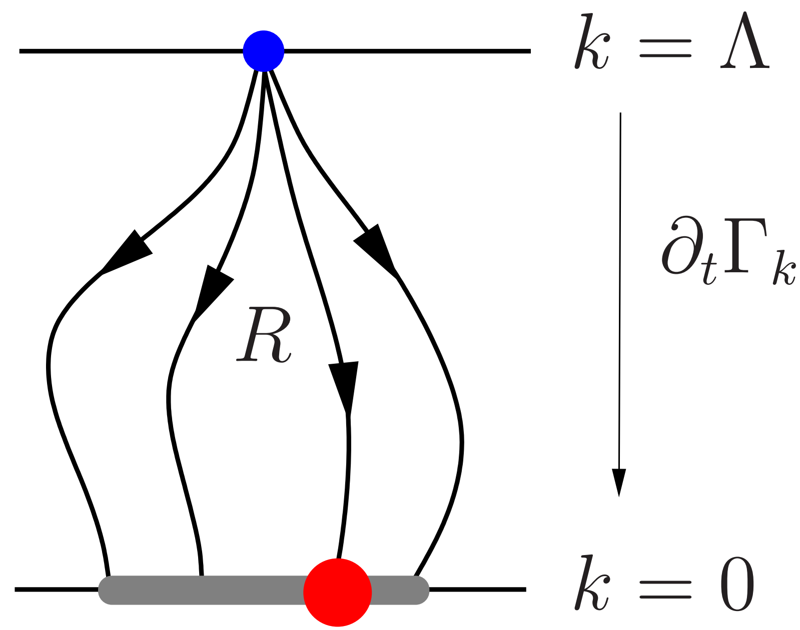


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- **optimisation**

  cutoff controls stability of flows
  $\longrightarrow$ identify most stable flows $\longrightarrow$ optimised cutoffs (DL '00, '01)

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- **modified Ward identities**

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$$\text{cutoff terms} \ \sim \langle D_\mu \frac{\delta}{\delta A_\mu} \Delta S_k \rangle \neq 0$$

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  **gauge invariant effective action**

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- **thermal flows for QCD**

  projection on thermal fluctuations
  axial gauge
  $\longrightarrow$ gauge-**invariant** flow for $\Gamma_k[T] - \Gamma_k[T=0]$
  (with J.M.Pawlowski)

# confinement in Landau gauge

- **Kugo-Ojima scenario**

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$$\langle A(p)A(-p)\rangle\Big|_{p^2\ll\Lambda^2_{\rm QCD}} \sim p^{-2(1-2\kappa)}$$

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- **Gribov-Zwanziger scenario**

  entropy, enhancement of ghost correlations $\longrightarrow \kappa > 0$.

# confinement in Landau gauge

- **Schwinger-Dyson eqs**
  infrared coefficients $\kappa > 0$, $\alpha_s$
  inclusion of quarks
  problems: RG scaling and renormalisation
- **stochastic quantisation**
  infrared coefficients $\kappa > 0$, $\alpha_s$
  resolution of Gribov problem
  problems: RG scaling and renormalisation
- **lattice**
  infrared behaviour of gluon propagator, ghost-gluon vertex
  problem: finite size scaling
- **flow equation**
  heavy quark potential, effective quark interacions
  problem: access to infrared regime

# propagator flows in Landau gauge

with J.M.Pawlowski, S. Nedelko, L.v.Smekal, PRL (2004)

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- **truncation**

  **general ghost and gluon two-point functions**

$$\Gamma_k^{(2)}(p^2) = z \cdot x^\kappa (1 + \delta Z(x)) \cdot p^2 \qquad (x = p^2/k^2)$$

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- **truncation**

  general ghost and gluon two-point functions

  vertices from Slavnov Taylor identities

    validity confirmed on the lattice (Cucchieri, Mendes, Mihara '04)

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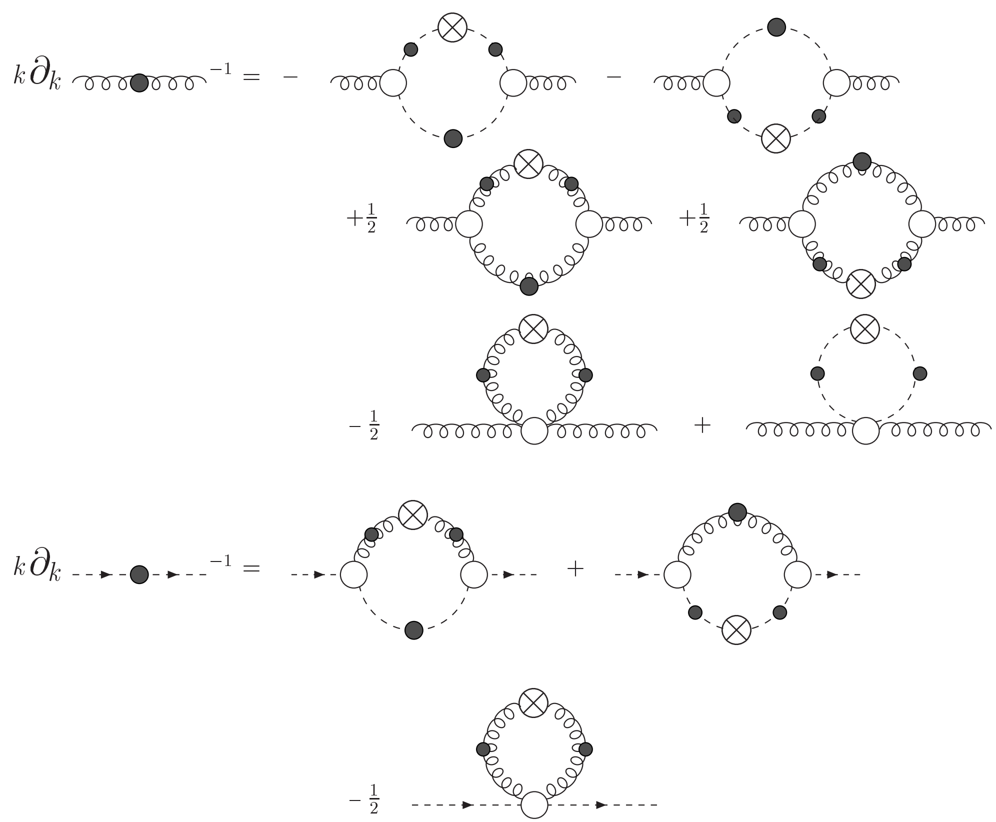

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$$k^2 \ll p^2 \ll \Lambda^2_{\rm QCD}$$

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physics already 'integrated-in'

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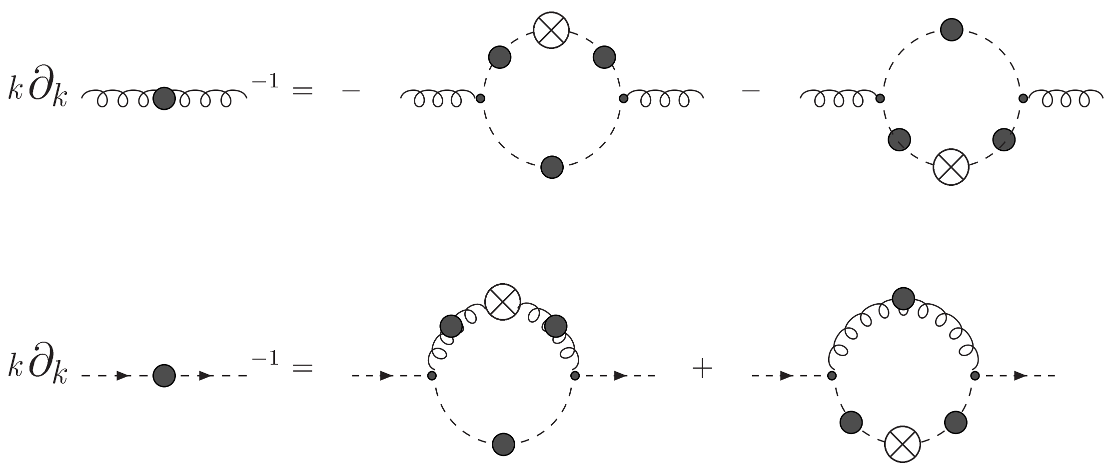

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$$k^2 \ll p^2 \ll \Lambda^2_{\rm QCD}$$

- **integration of flow**

$$\delta Z(x) = \frac{\alpha_s}{\pi^2} N \int_x^\infty \frac{dx'}{x'^{2+\kappa}}\, f(x'; x, \kappa, \delta Z; R)$$

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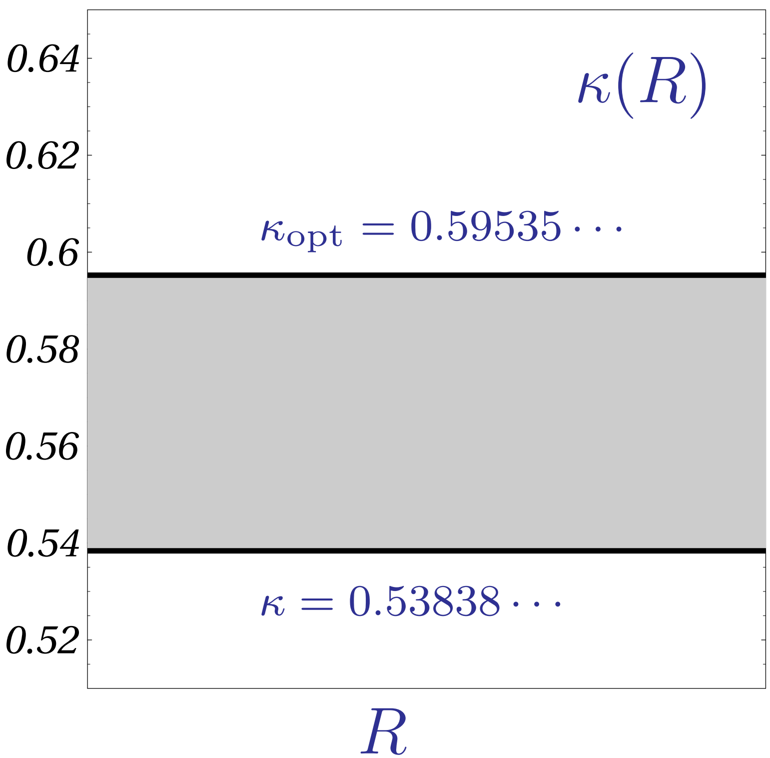
- **integration of flow**

  deduce infrared coefficient $\kappa$

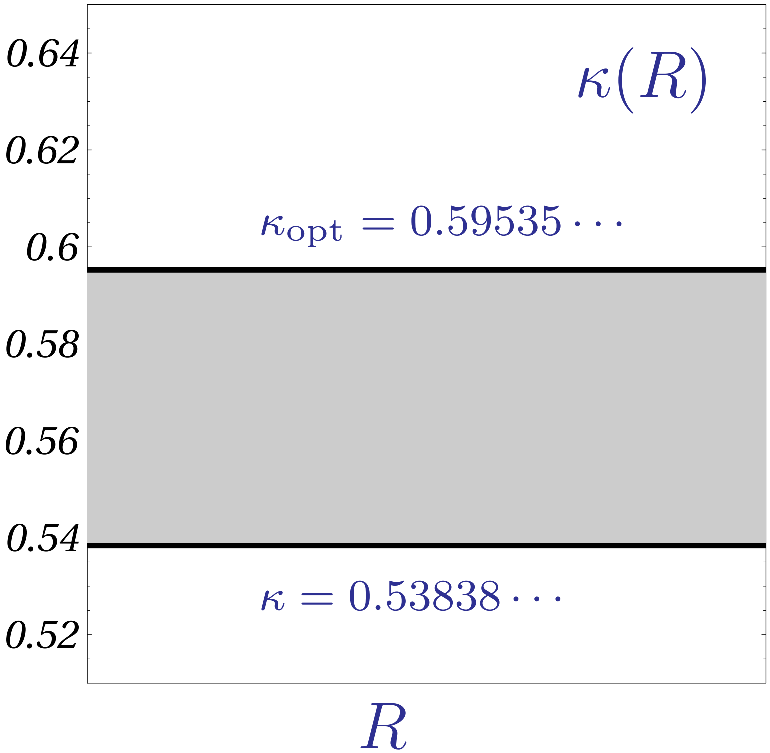
  Landau gauge is RG fixed point (DL, Pawlowski '98)

# results



0.64
0.62
0.6
0.58
0.56
0.54
0.52
$\kappa(R)$
$\kappa_{\rm opt} = 0.59535\cdots$
$\kappa = 0.53838\cdots$
$R$

# results




## iterative solution

- iteration in $\delta Z(x)$

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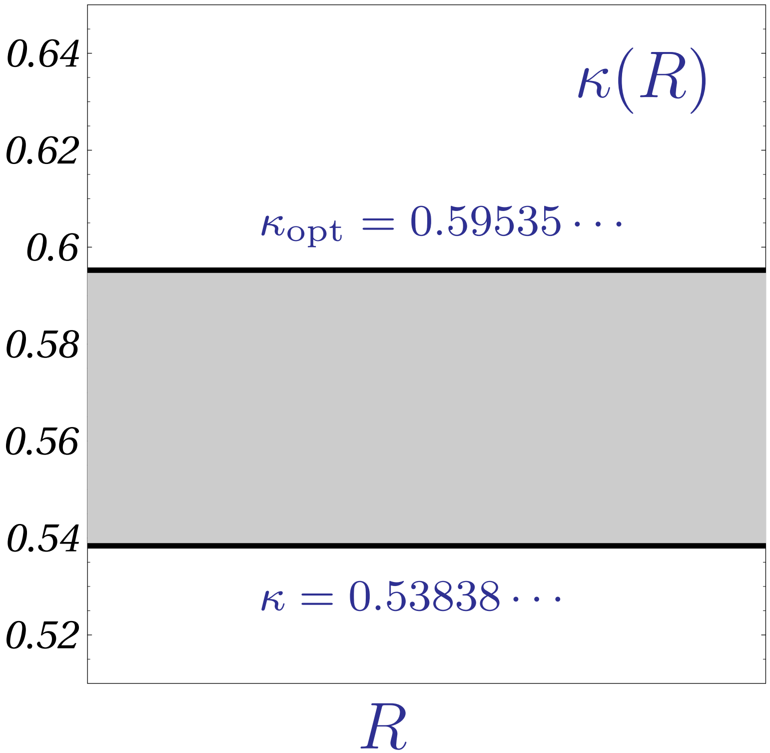


## iterative solution

- LO is cutoff independent

$$\kappa(R) = \kappa_{\mathrm{opt}}$$

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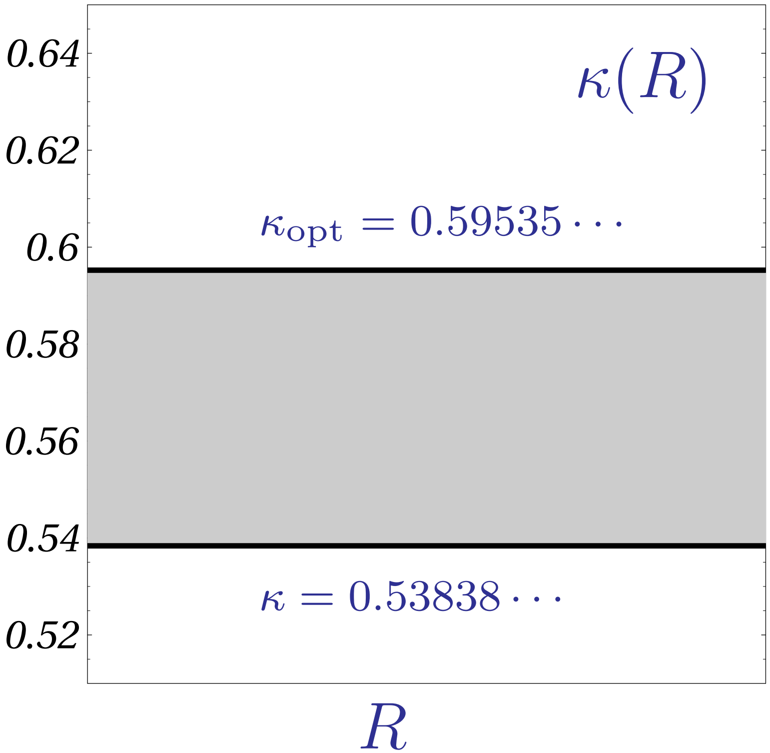


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$$\kappa_{\text{opt}} = \text{extr}_R\, \kappa(R)$$

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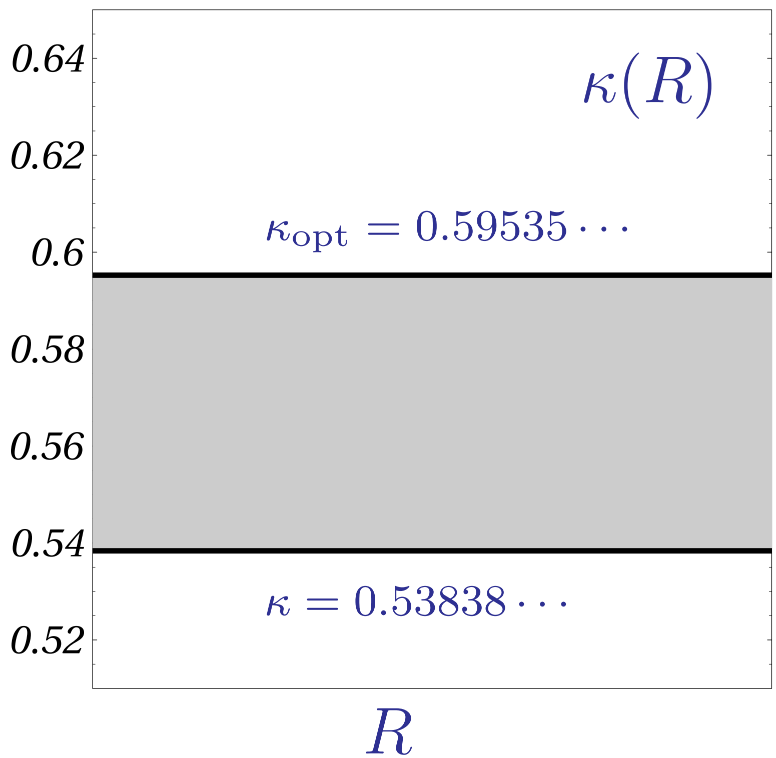


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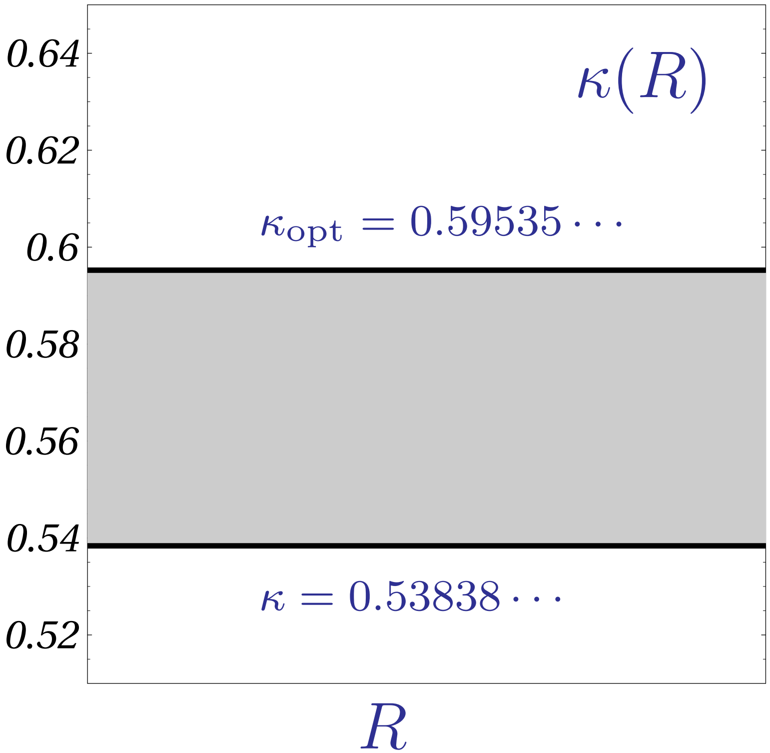


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## consistency

- DS result (Lerche, Smekal '02)

$$\kappa_{\rm DS} = 0.59535$$
$$\alpha_s = 2.9717$$

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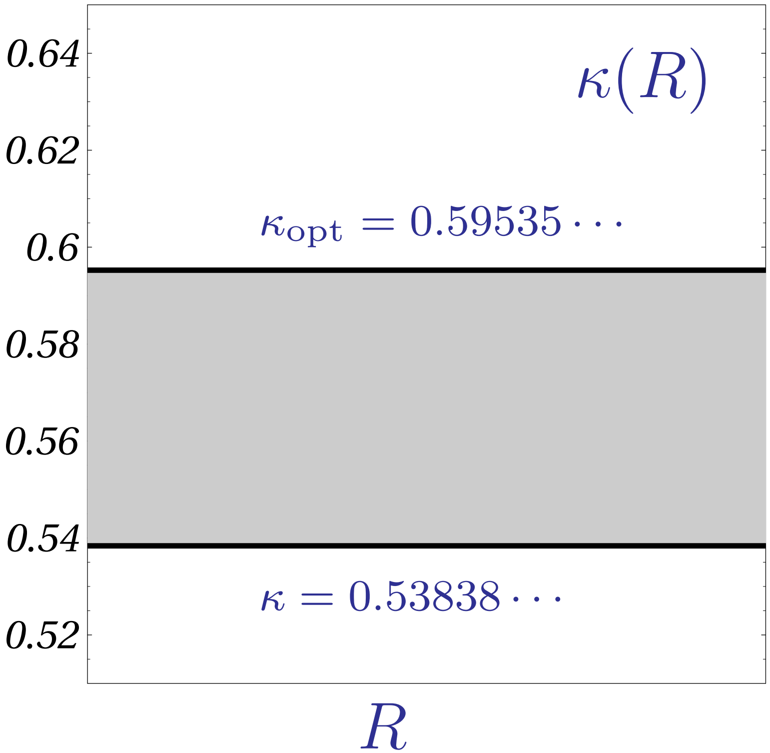


**iterative solution**

- LO is cutoff independent
- beyond LO: global extrema
- optimisation leads to $\kappa_{\text{opt}}$

**consistency**

- $\kappa_{\text{opt}}$ coincides with DS result
- lattice simulations (Oliveira, Silva '04)

$$\kappa_{\text{lattice}} = 0.53 - 0.54$$

# conclusions and outlook

- **infrared regime of QCD**

  **fixed point behaviour, analytical access**
  **infrared coefficients $\kappa$ and $\alpha_s$**
  **signatures of confinement**

  **full vertex functions, fermions**

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- **finite density**

  inclusion of quarks, chemical potential