A generalization of the imaginary chemical potential method

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The method

Goal: get insight on the QCD phase diagram overcoming the sign problem

Chemical potential on a lattice appears through:

$$\begin{aligned} \exp(\mu) &= \cos(\mu) + \sinh(\mu) \\ \exp(-\mu) &= \cos(\mu) - \sinh(\mu) \end{aligned}$$

Consider two independent parameters, x and y:

$$\begin{aligned} \cosh(\mu) &\longrightarrow x \\ \sinh(\mu) &\longrightarrow y \end{aligned}$$

Fermion action

$$S = \frac{1}{2} \sum_n \sum_{i=1}^{3} \bar{\psi}_n \eta_i(n) \left(U_{n,i} \psi_{n+i} - U^\dagger_{n-i,i} \psi_{n-i} \right) + S_\tau$$

$$\begin{aligned} S_\tau &= x \, \tfrac{1}{2} \textstyle\sum_n \bar{\psi}_n \eta_0(n) \left(U_{n,0} \psi_{n+0} - U^\dagger_{n-0,0} \psi_{n-0} \right) \\ &+ y \, \tfrac{1}{2} \textstyle\sum_n \bar{\psi}_n \eta_0(n) \left(U_{n,0} \psi_{n+0} + U^\dagger_{n-0,0} \psi_{n-0} \right) \end{aligned}$$

No sign problem if $y = \mathrm{i}\,\bar{y}$, $\bar{y}$ real

- ⋆ QCD: $x = \cosh(\mu)$, $y = \sinh(\mu)$
- ⋆ Imaginary μ: $x = \cos(\mu)$, $y = \mathrm{i}\,\sin(\mu)$

Symmetries

The partition function $\mathcal{Z}(x, y)$ is invariant under

$$\boxed{\begin{aligned} x' &= \cos(\theta_k)\, x \;+\; \mathrm{i}\sin(\theta_k)\, y \\ y' &= \mathrm{i}\sin(\theta_k)\, x \;+\; \cos(\theta_k)\, y \end{aligned}}$$

where $\exp(\mathrm{i}\theta_k) \in Z_{3L_t}$.

Together with CPT $\Longrightarrow$ $\mathcal{Z}(u, v)$, with

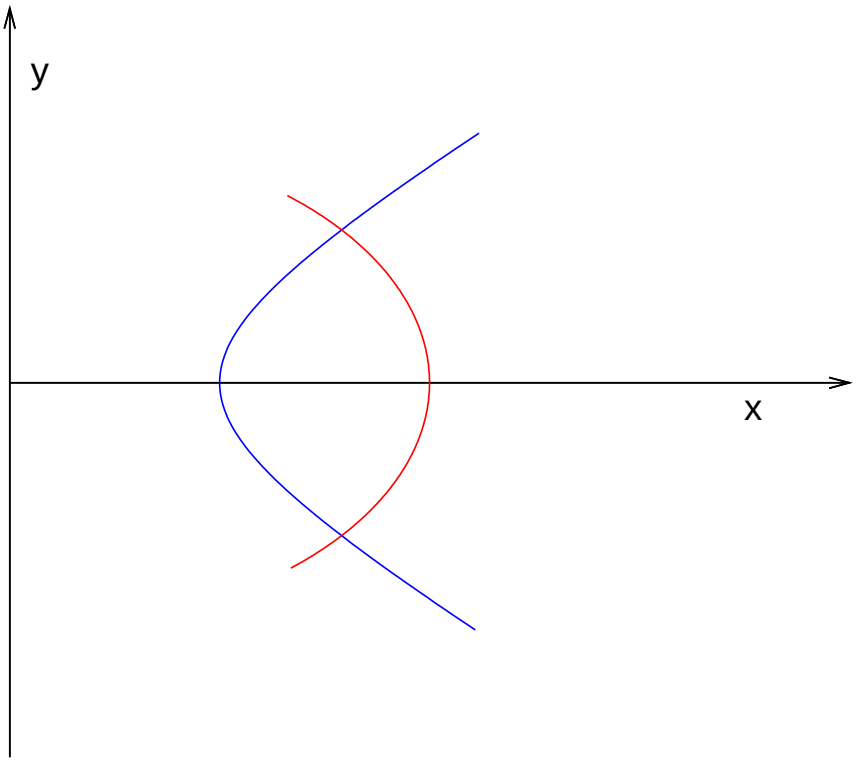
$$\boxed{\begin{aligned} u &= x^2 - y^2 \\ v &= (x+y)^{3L_t} + (x-y)^{3L_t} \end{aligned}}$$

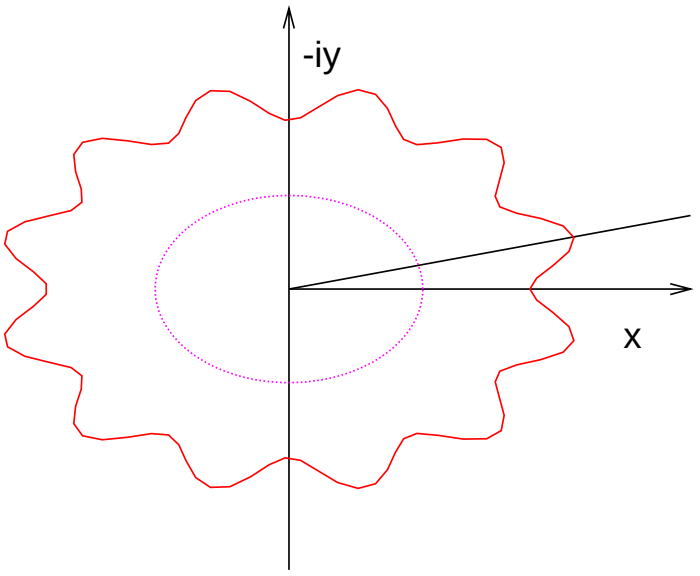
At imaginary $y = \mathrm{i}\,\bar{y}$

$$\begin{aligned} u &= \rho^2 & \qquad \rho^2 &= x^2 + \bar{y}^2 \\ v &= \rho^{3L_t}\cos(3L_t\eta) & \qquad \eta &= \arctan(\bar{y}/x) \end{aligned}$$

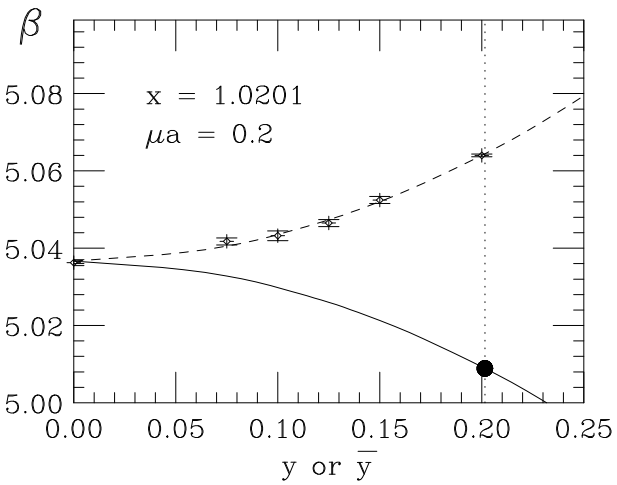
Hence, $\mathcal{Z}$ is $2\pi/3L_t$ periodic in $(x,\, \bar{y})$ plane

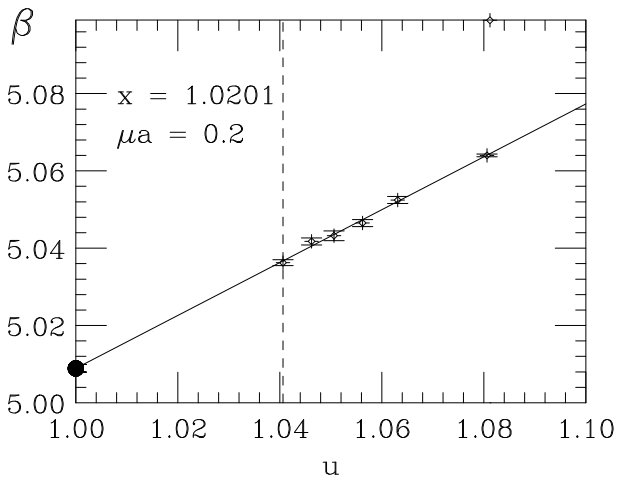
Expected phase diagram

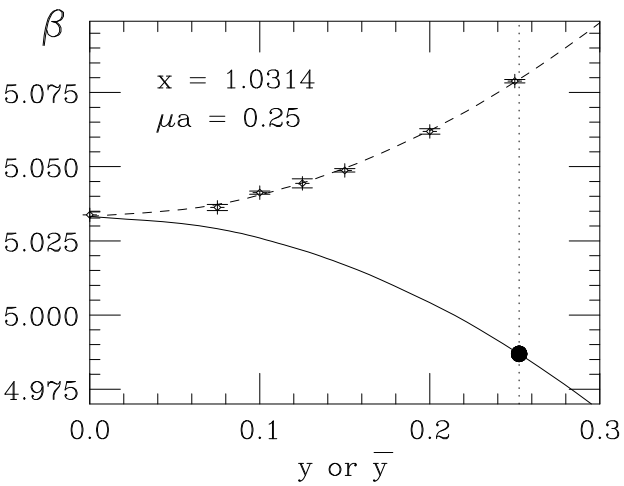


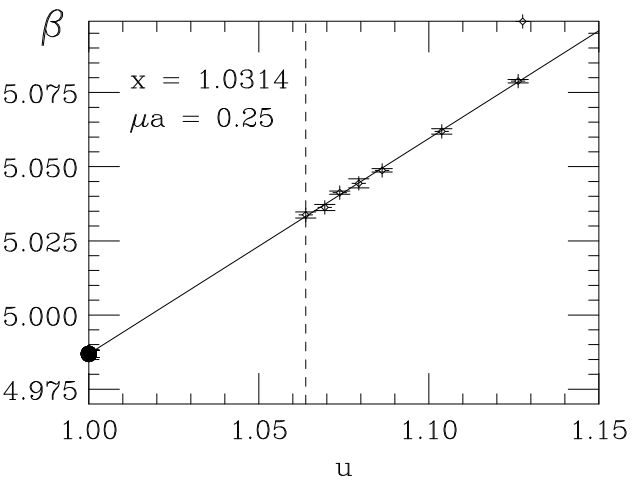


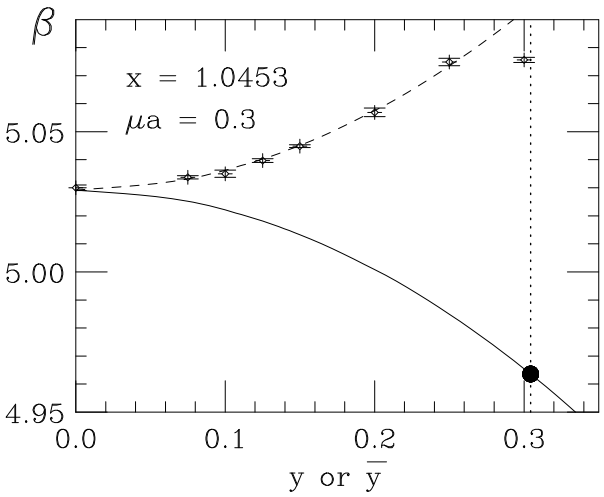
Results in 4-flavor QCD

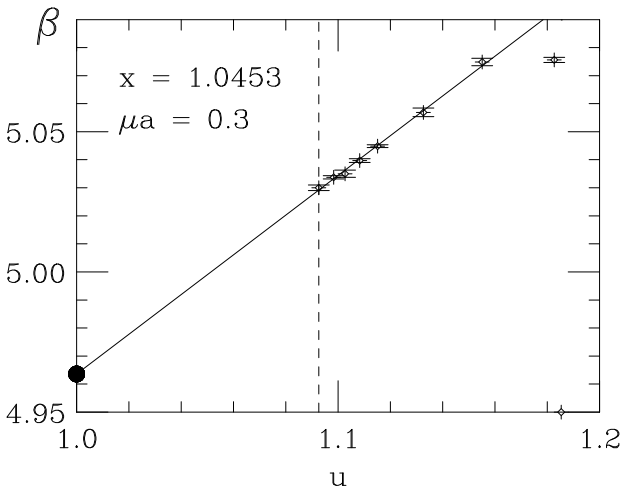


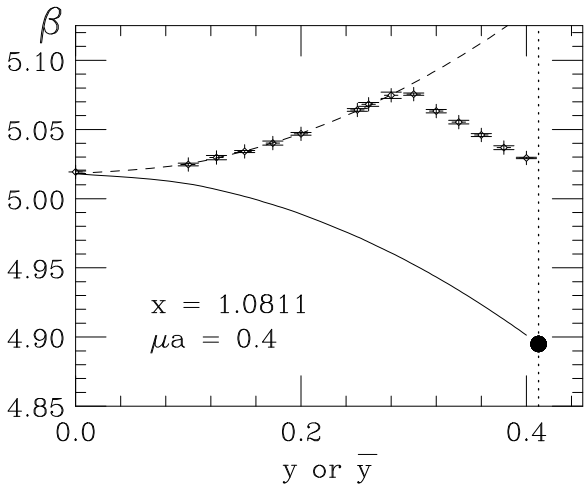


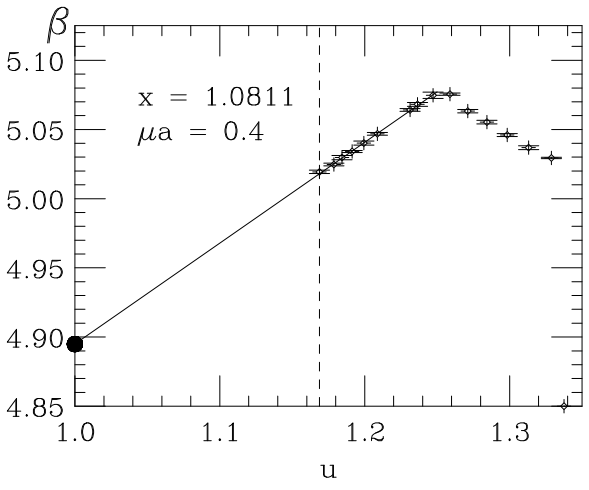


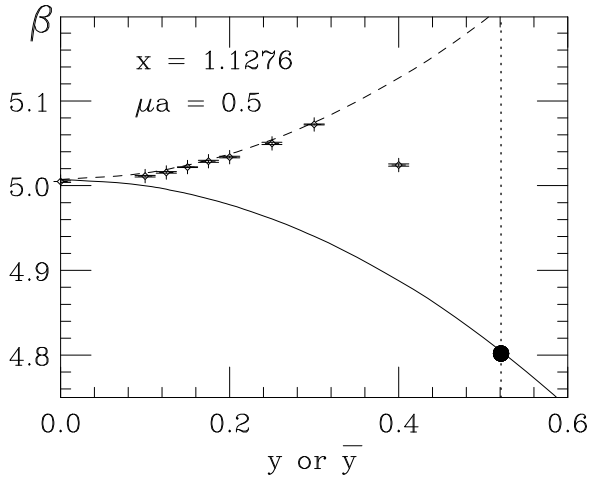


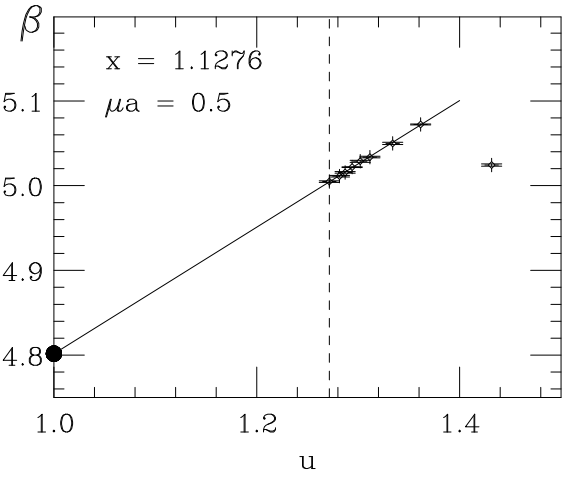

β
x = 1.0453
μa = 0.3
5.05
5.00
4.95
0.0
0.1
0.2
0.3
y or $\bar{y}$


β
x = 1.0453
μa = 0.3
5.05
5.00
4.95
1.0
1.1
1.2
u

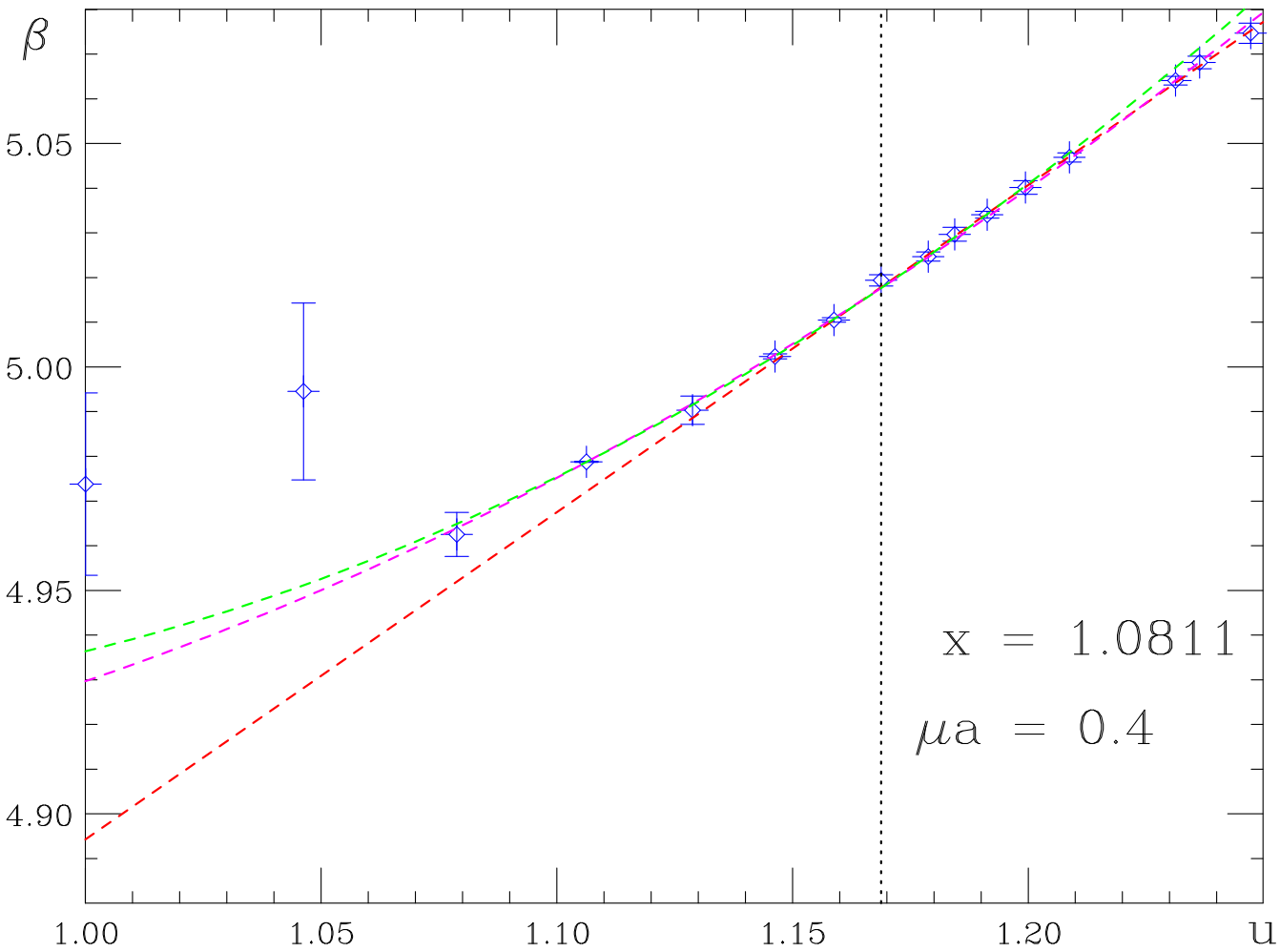

β
5.10
5.05
5.00
4.95
4.90
4.85
x = 1.0811
μa = 0.4
0.0
0.2
0.4
y or $\bar{y}$


β
5.10
5.05
5.00
4.95
4.90
4.85
x = 1.0811
μa = 0.4
1.0
1.1
1.2
1.3
u

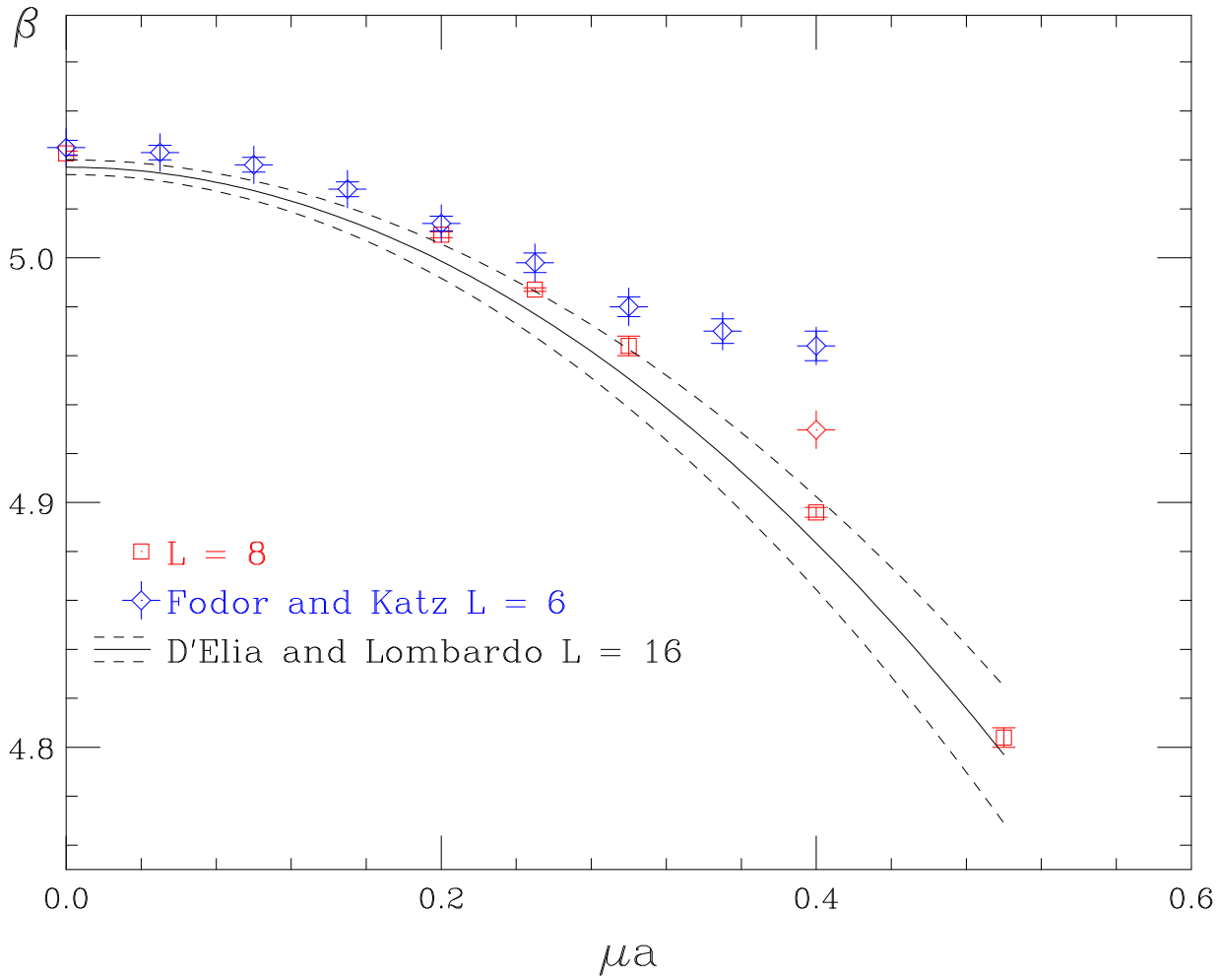

β
x = 1.1276
μa = 0.5
5.1
5.0
4.9
4.8
0.0
0.2
0.4
0.6
y or $\bar{y}$


β
x = 1.1276
μa = 0.5
5.1
5.0
4.9
4.8
1.0
1.2
1.4
u

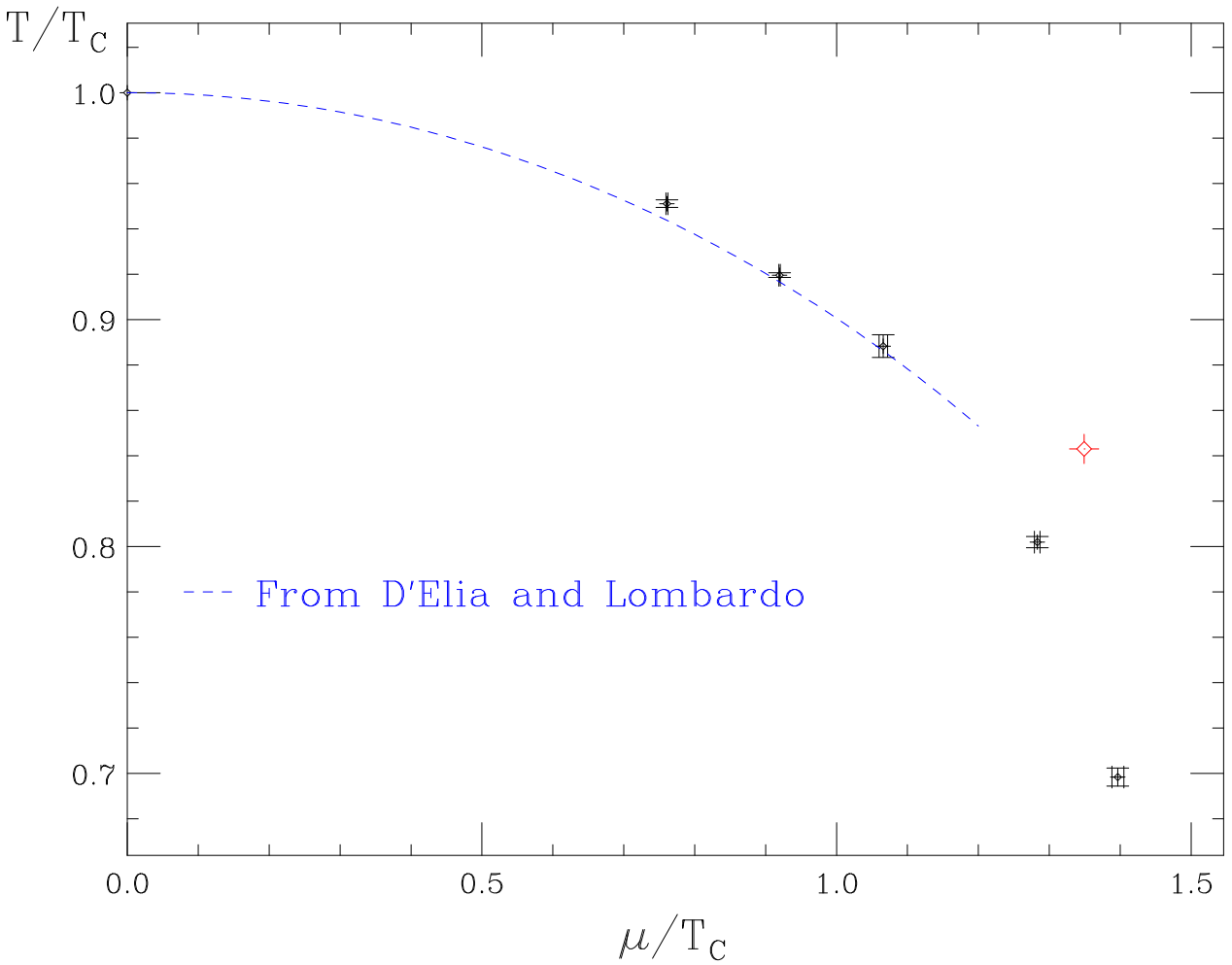
Use double reweighting to penetrate into the forbidden region



Physical phase diagram

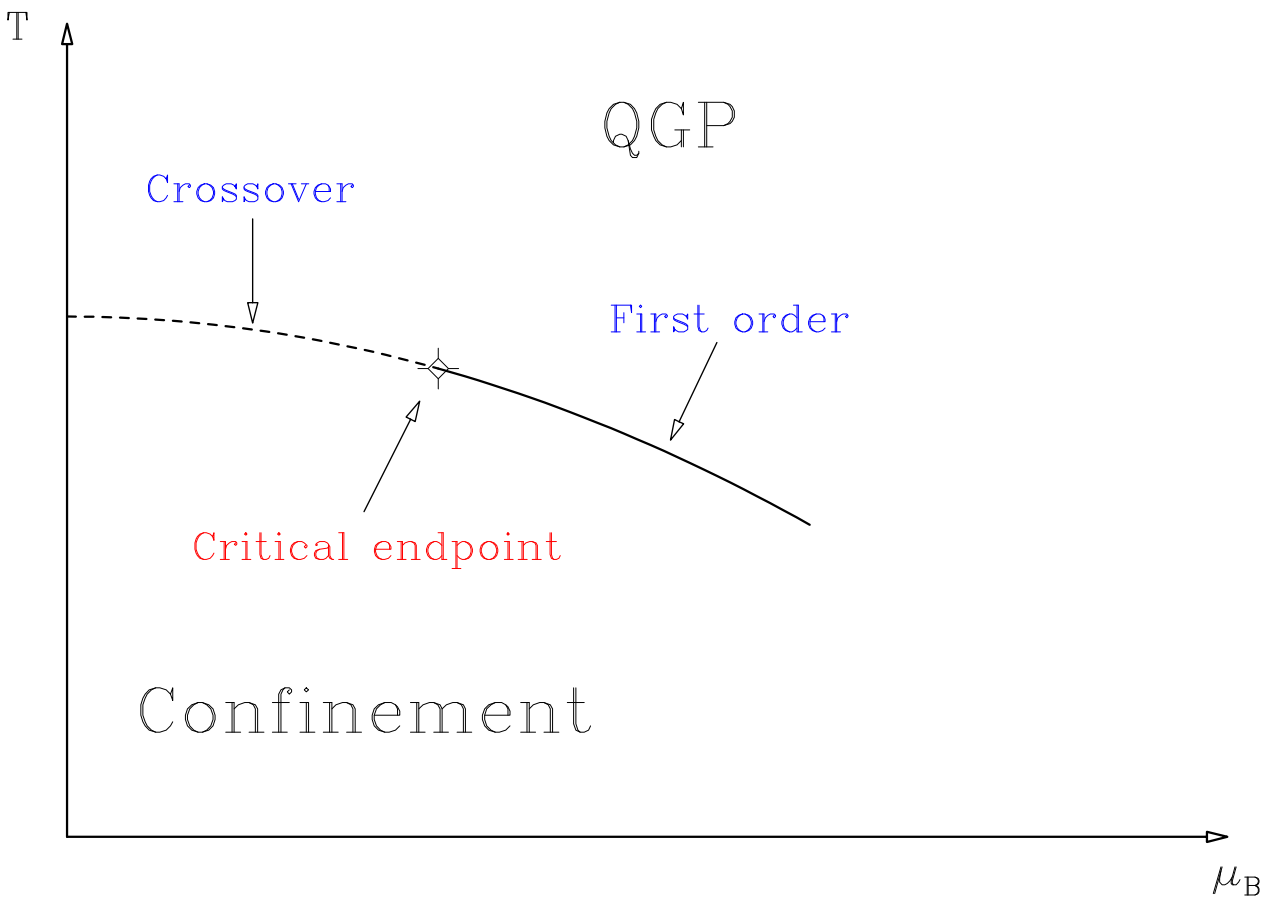


Physical phase diagram



Critical endpoint in 2-flavour QCD

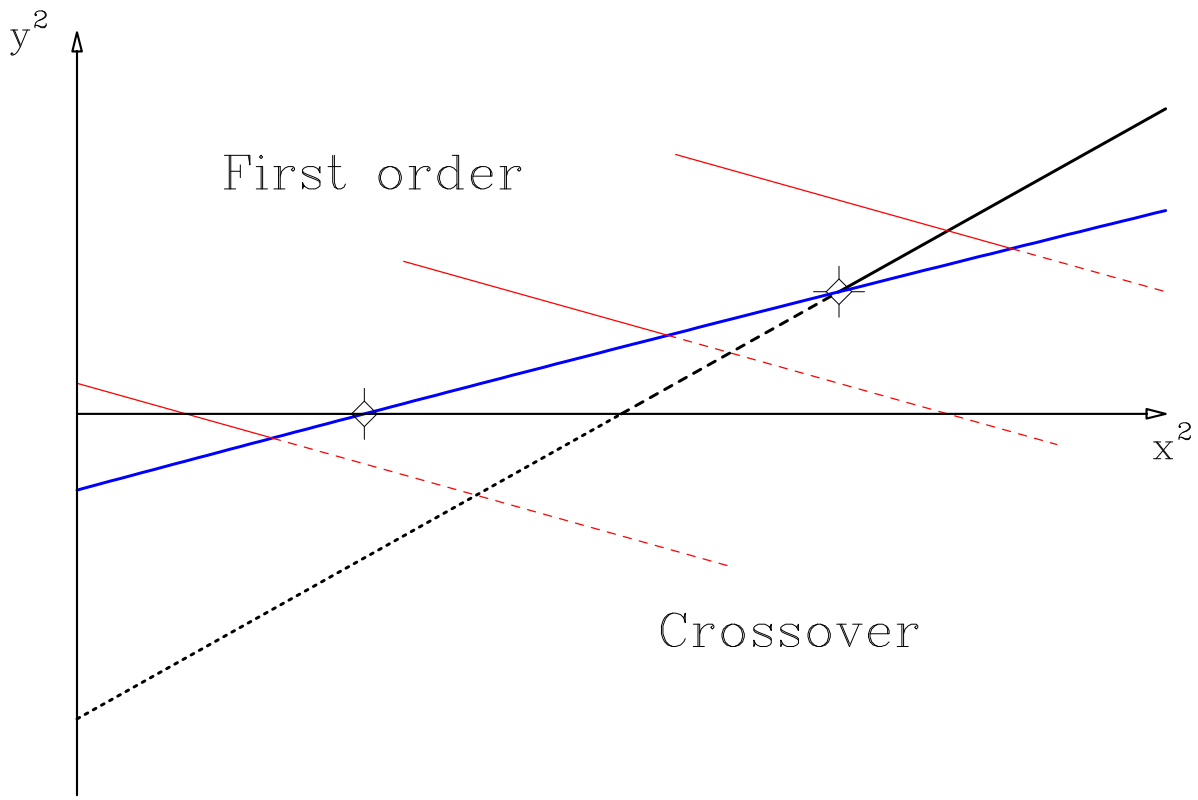
The expected phase diagram of QCD



It is expected:

- ⋆ a crossover at $\mu = 0$
- ⋆ a first order transition at large enough μ

Expected phase diagram in the (x^2, y^2) plane



- ⋆ At $x^2 = y^2 = 0$ the Polyakov loop is an order parameter. First order transition expected.
- ⋆ At $x^2 = 1$ and $y^2 = 0$ no transition (crossover)
- ⋆ For $y^2 = 0$ the first order transition disappears at a critical point at $x^2 = x_c^2 > 0$
- ⋆ For $y^2 \neq 0$ critical point $\Longrightarrow$ critical line
- ⋆ The critical line can be computed for $y^2 \leq 0$
- ⋆ Extrapolate the critical line to the physical line

A Random Matrix Model

A CRMM adapted to the lattice:

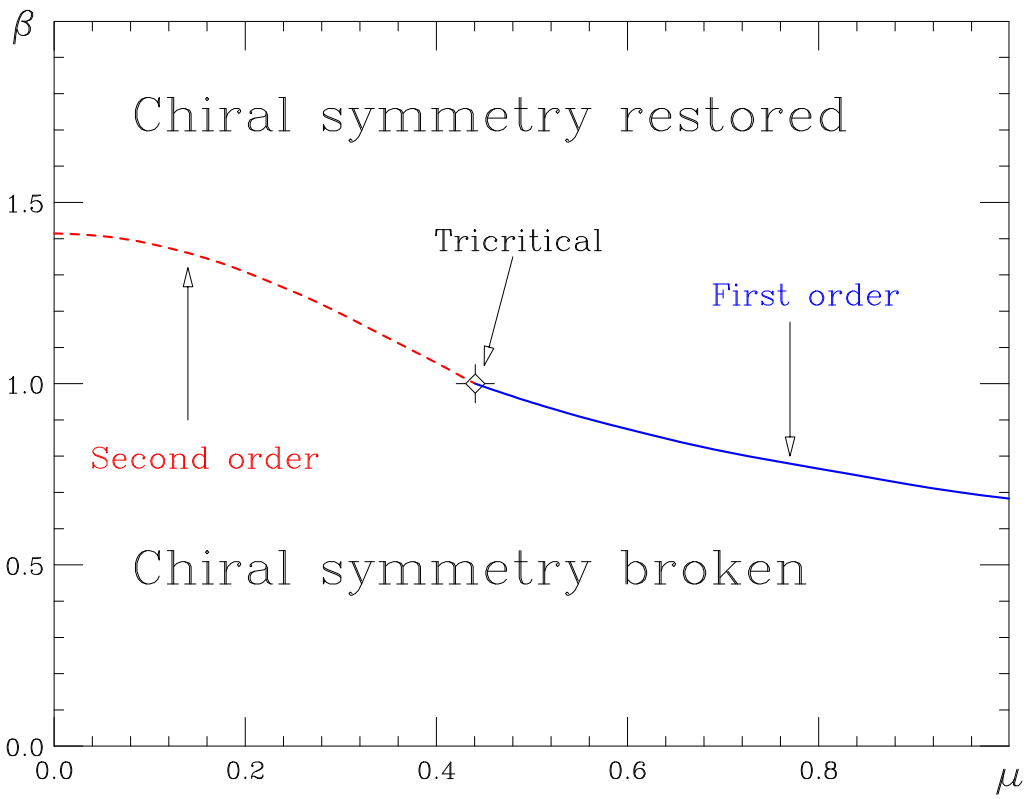
$$\mathcal{Z} = \int [dA^\dagger dA][d\bar{\psi}_L d\psi_L d\bar{\psi}_R d\psi_R] \exp(-S)$$

$$\begin{aligned}
S &= N \sum_{t=0}^{L_t-1} \mathrm{Tr}(A_t^\dagger A_t) \\
&- \sum_{t=0}^{L_t-1} \left[\bar{\psi}_L^a(t) \mathrm{i} A_t \psi_L^a(t) + \bar{\psi}_R^a(t) \mathrm{i} A_t^\dagger \psi_R^a(t)\right] \\
&- m \sum_{t=0}^{L_t-1} \left[\bar{\psi}_L^a(t)\psi_R^a(t) + \bar{\psi}_R^a(t)\psi_L^a(t)\right] \\
&- \beta \frac{x+y}{2} \sum_{t=0}^{L_t-1} \left[\bar{\psi}_L^a(t)\psi_L^a(t+1) + \bar{\psi}_R^a(t)\psi_R^a(t+1)\right] \\
&- \beta \frac{x-y}{2} \sum_{t=0}^{L_t-1} \left[\bar{\psi}_L^a(t+1)\psi_L^a(t) + \bar{\psi}_R^a(t+1)\psi_R^a(t)\right]
\end{aligned}$$

A_t: $N \times N$ matrix; a: flavour index

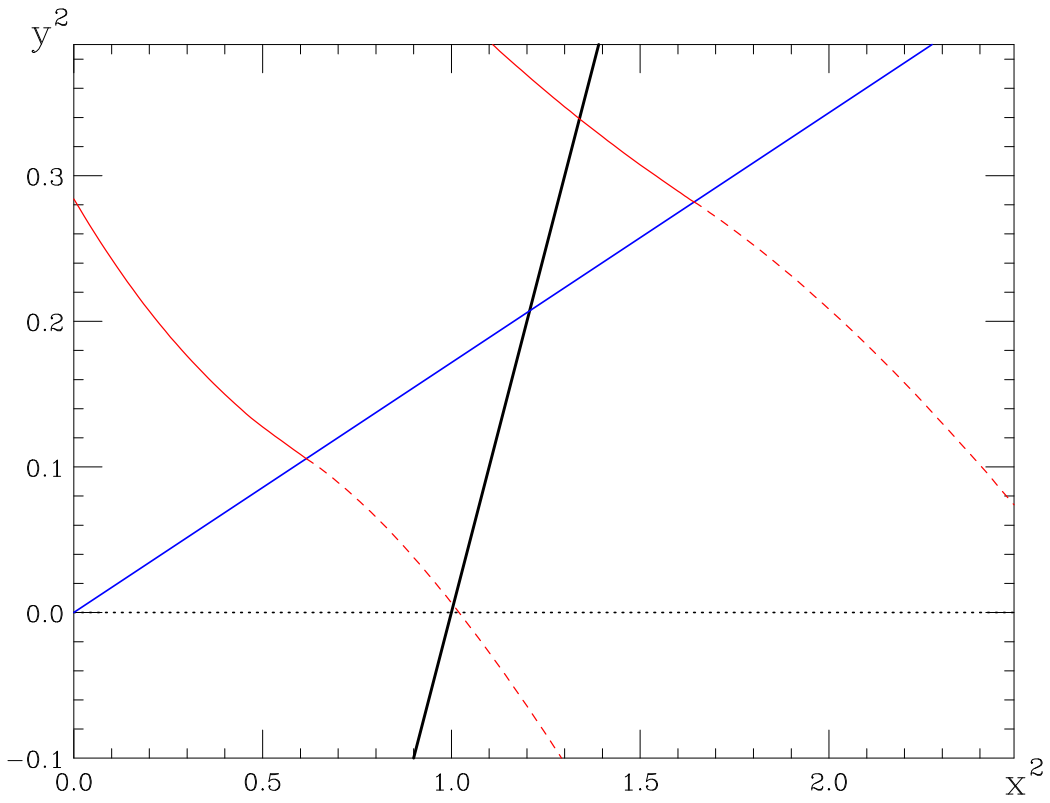
Physical phase diagram

$x = \cosh(\mu)$ $\quad$ $y = \sinh(\mu)$



Reproduces the same phase diagram as standard CRMM

Phase diagram in the (x^2,y^2) plane



Tricritical line:

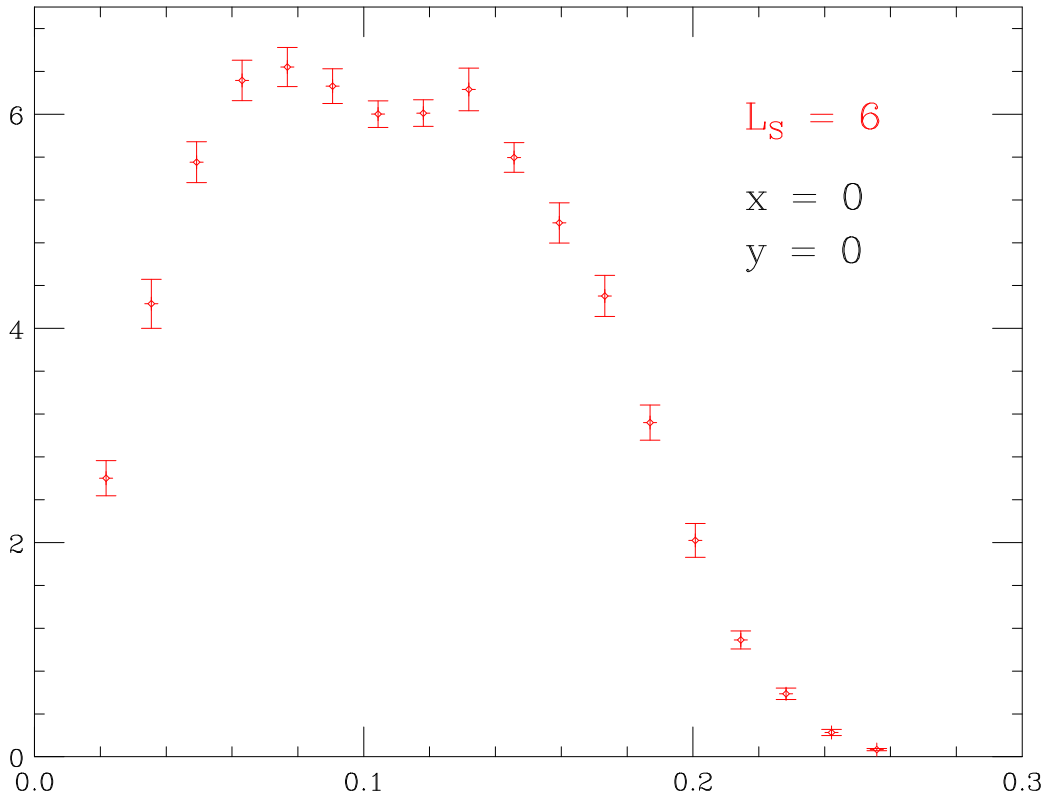
$$\begin{aligned} x_t^2(y) &= (3+2\sqrt{2})\, y^2 \\ \beta_t^2(y^2) &= \tfrac{\sqrt{2}-1}{2}\, y^{-2} \end{aligned}$$

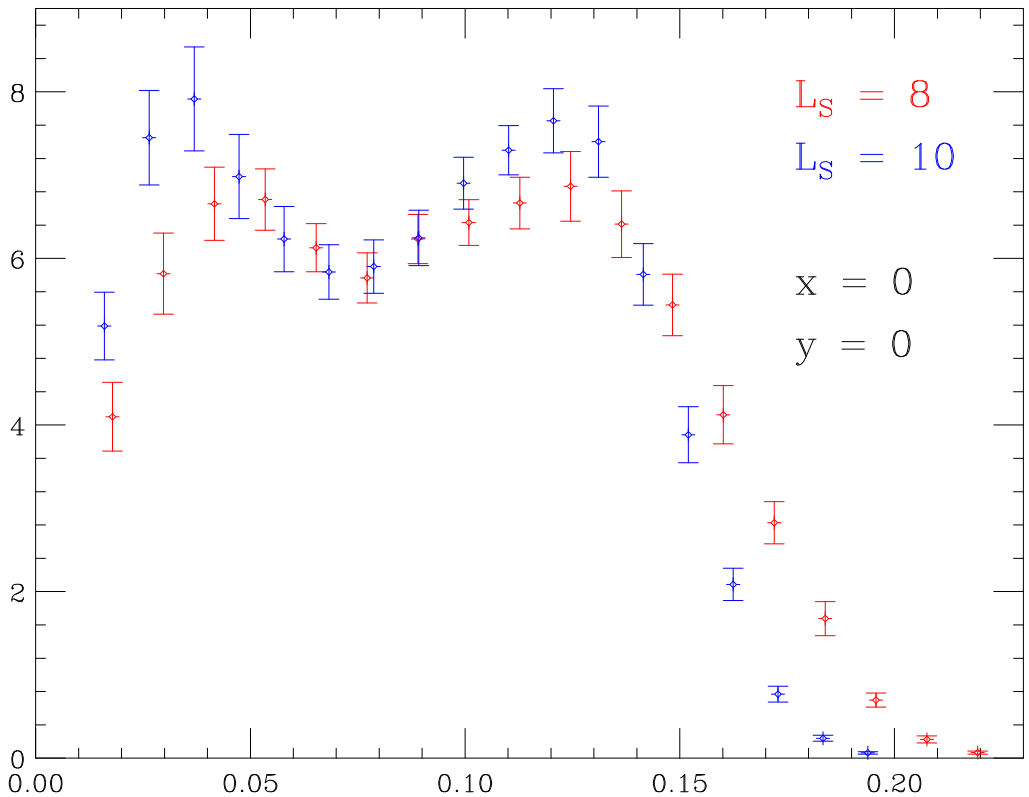
The $x = y = 0$ point is singular

Pathology of the model

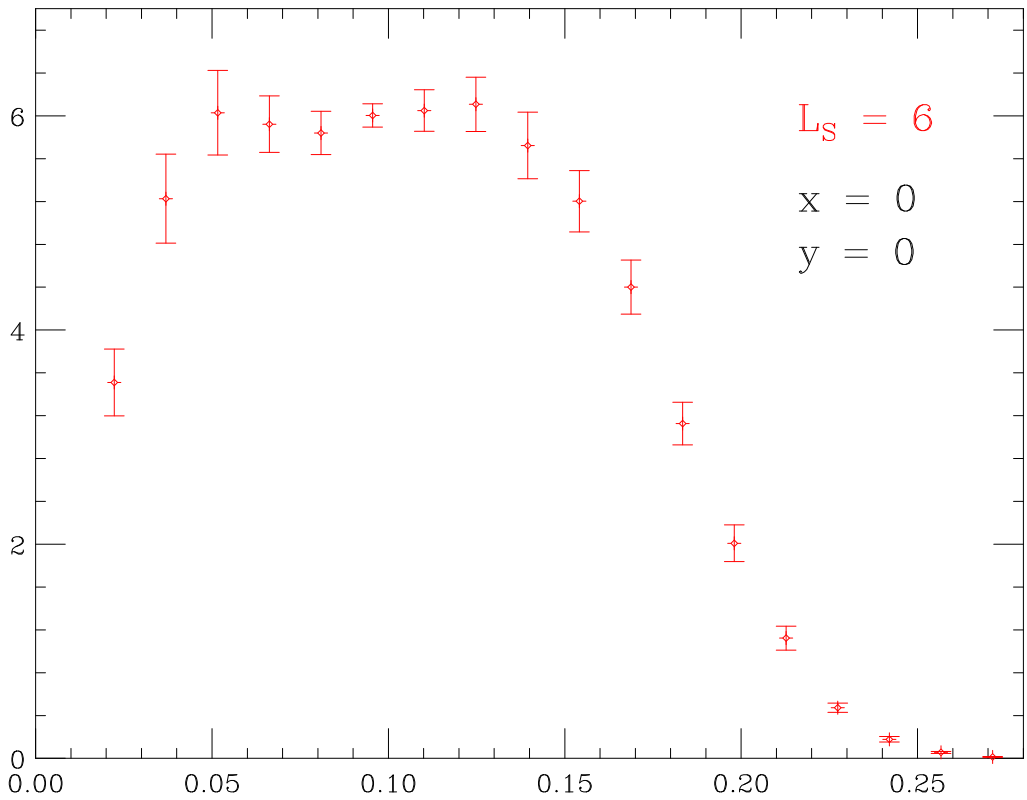
Results in QCD for $y = 0$

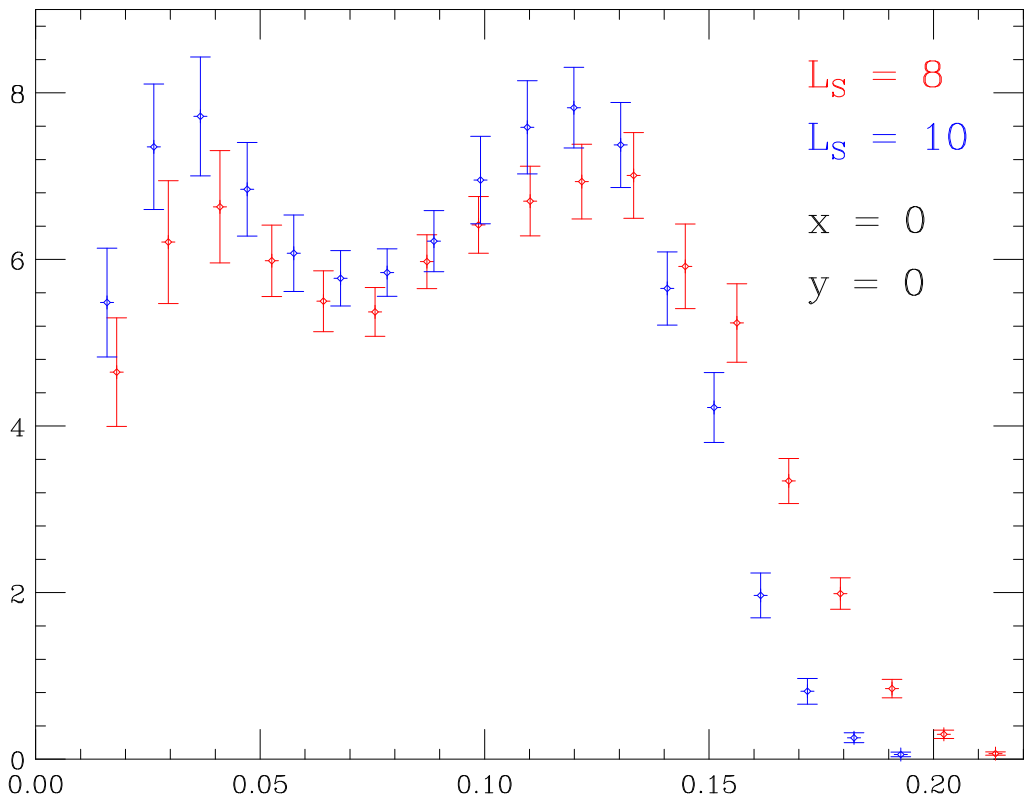
At $x = 0$ and $y = 0$: first order transition



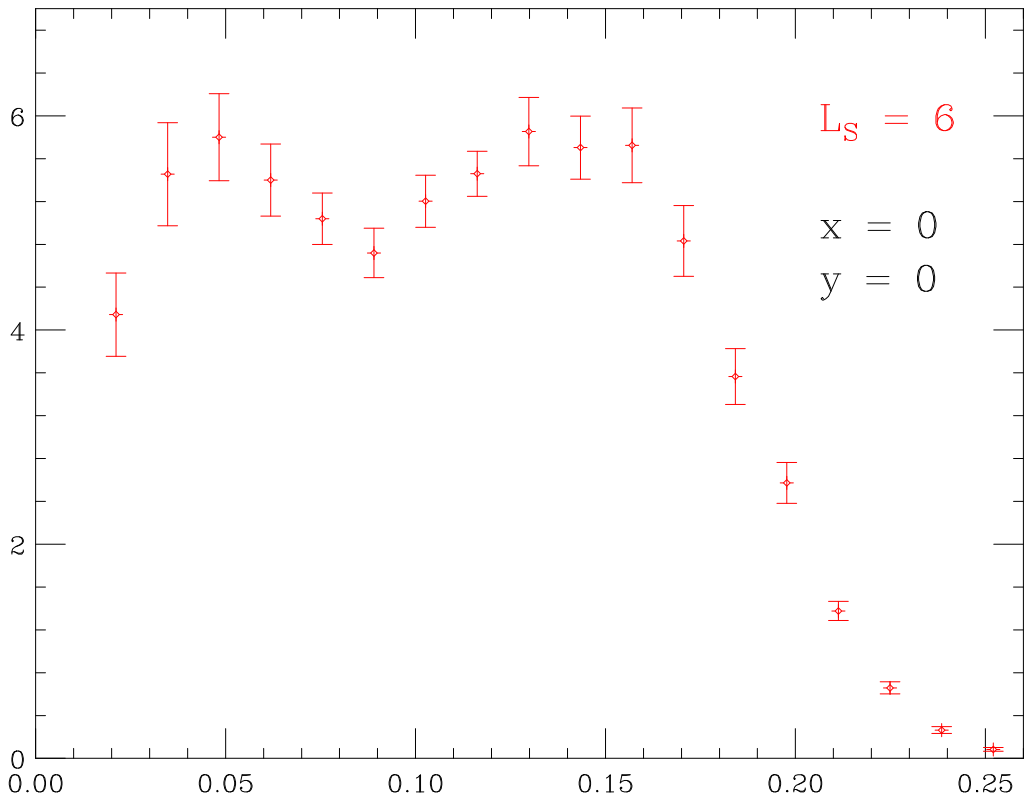


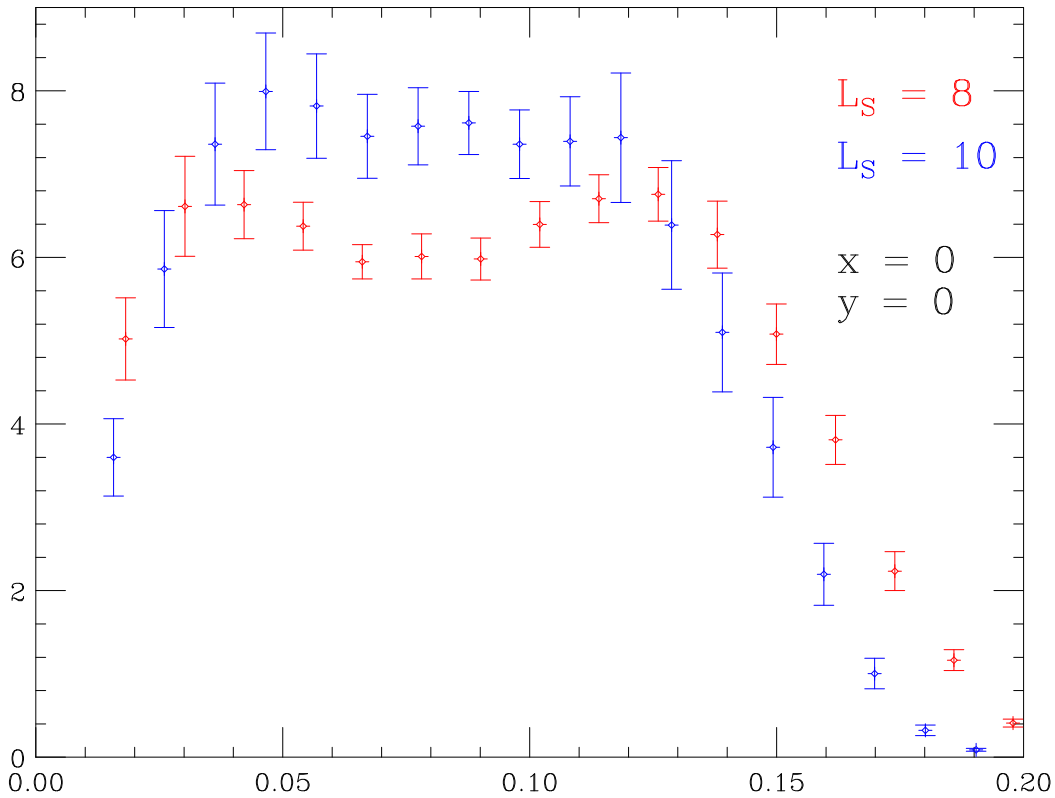
At $x = 0.5$ and $y = 0$ the transition looks first order also:



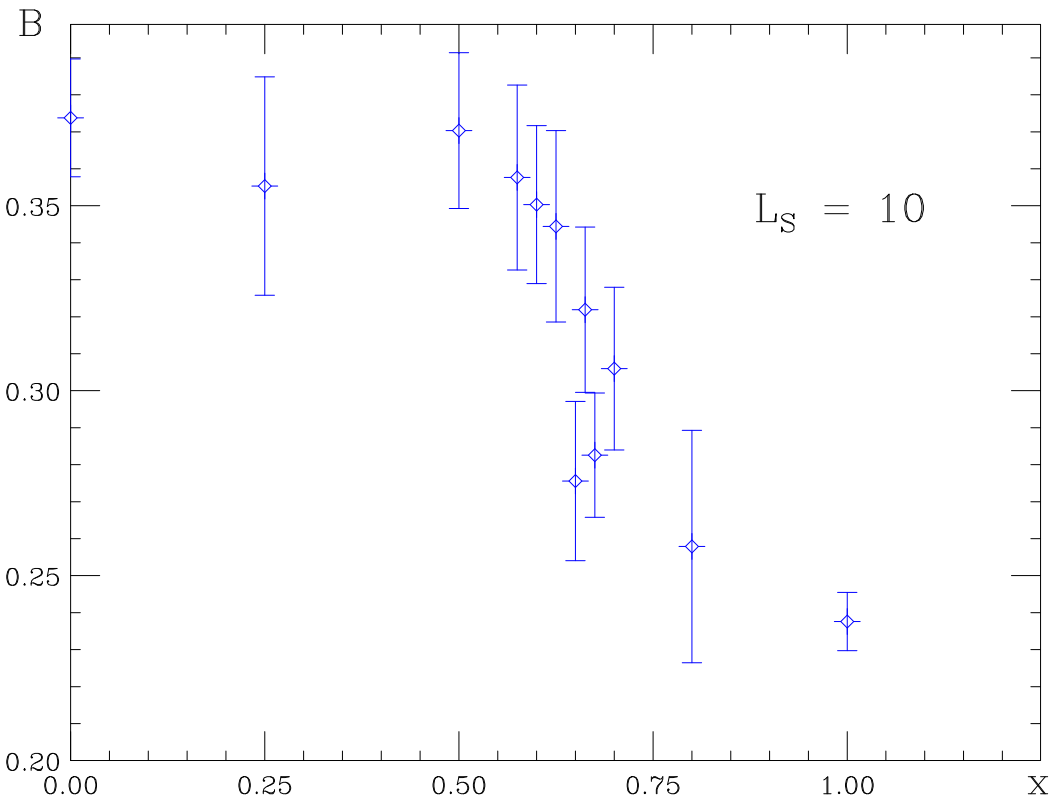


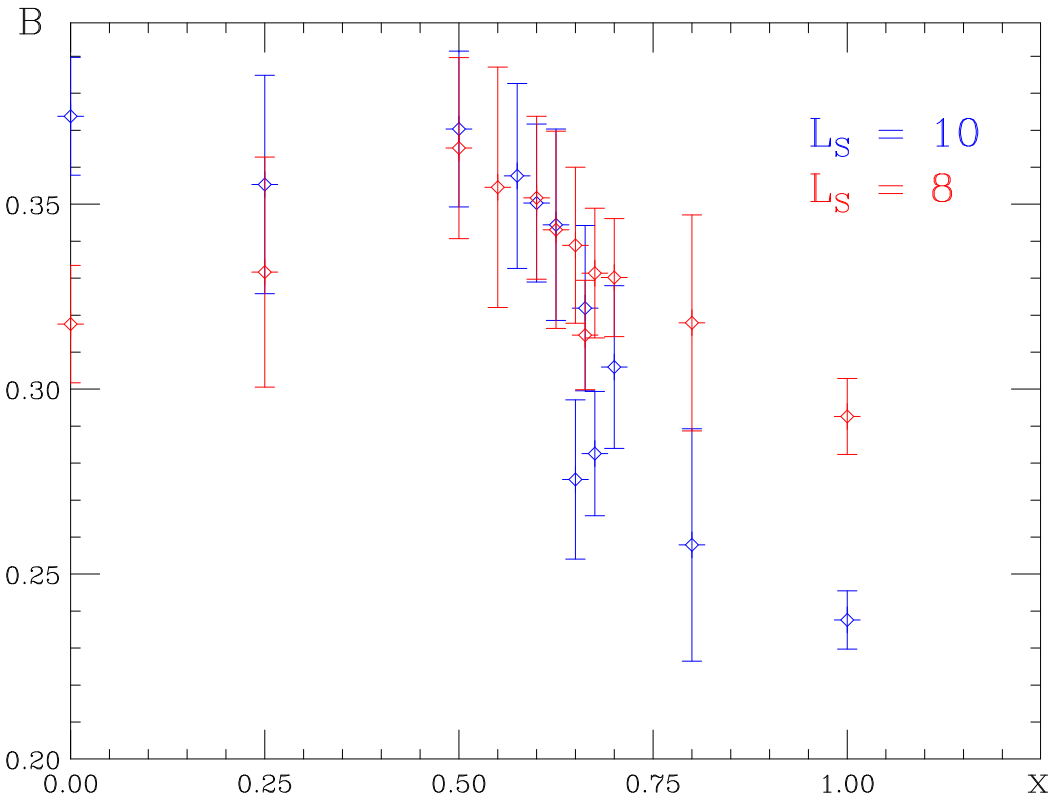
At $x = 0.7$ and $y = 0$ the transition looks like a crossover:





Binder cumulant





Critical point at $x_c \approx 0.6 \quad (y = 0)$