Lattice QCD at finite density

Ph. de Forcrand[1,2]
with
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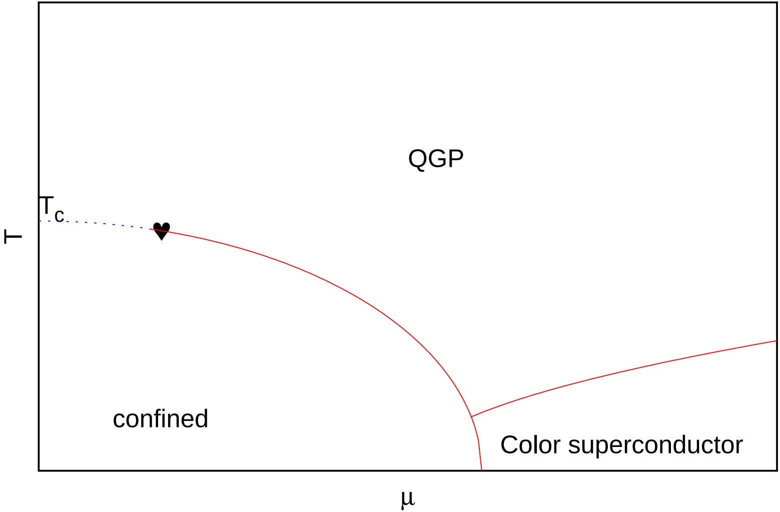
[5]Univ. Hiroshima

Outline

Expectations



To be checked by lattice QCD simulations

The difficulty: “sign” problem

γ_5-hermiticity:

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BUT $\gamma_5(i\not p + m + \mu\gamma_0)\gamma_5 = (-i\not p + m - \mu\gamma_0) = (i\not p + m - \mu\gamma_0)^*$

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$Z(\mu) = \int \mathcal{D}U\, e^{-S_g} \det{}^{N_f} \not D(\mu) \quad \rightarrow$ no Monte Carlo

$Z_{MC} = \ldots |\det|$ or $\det(\mu = 0)$ or ...

$Z(\mu)/Z_{MC} \sim \exp(-V\delta F(\mu)) \quad \rightarrow$ exponential in V

$\Rightarrow$ small μ

Ask SIMPLE questions: (ie. derivatives at $\mu = 0$)

- physically relevant: RHIC
- can address fundamental issue in phase diagram

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Simulations at $\mu = 0$

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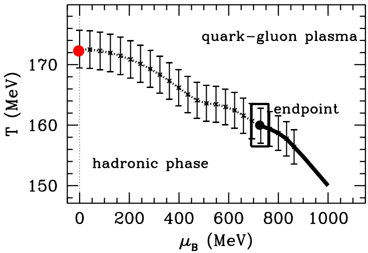
Numerical approaches

I. Reweighting in (μ, β) from $(\mu = 0, \beta_c)$

Fodor & Katz

$$Z(\mu,\beta) = \left\langle \frac{\exp(-\beta S_g)\det M(\mu)}{\exp(-\beta_c S_g)\det M(\mu=0)} \right\rangle Z_{MC}(\mu=0,\beta_c)$$



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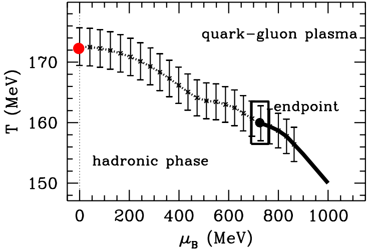

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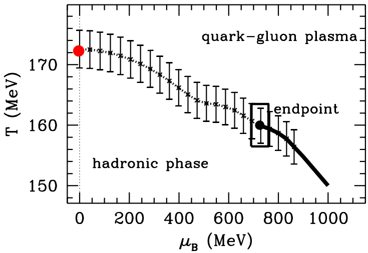

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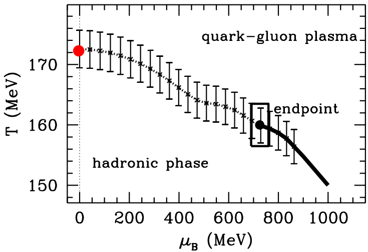

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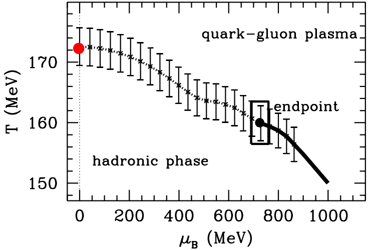

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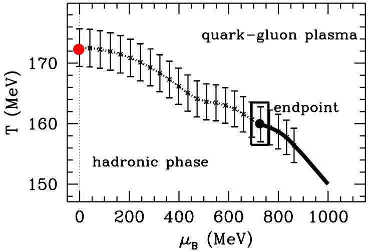

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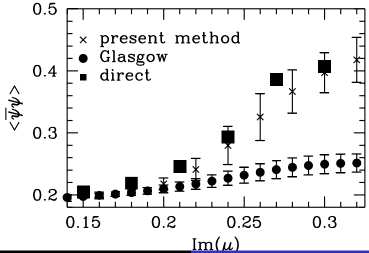

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Bielefeld-Swansea I

Ib. Reweighting + density of states

$Z = \int dx \int \mathcal{D}U \exp(-\beta S_g) \det M(\mu) \delta(P - x)$

increase statistics as needed for each $x \longrightarrow$ larger μ, lower T

Takaishi; Fodor, Katz & Schmidt

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MILC, .., TARO, Bielefeld-Swansea II, Gavai & Gupta

A few derivatives (max. 4); convergence?

Choose m_q, look for non-analyticity at critical point ?

III. Imaginary μ + analytic continuation

PdF & OP, D'Elia & Lombardo, Giudice & Papa, Chen & Luo, Azcoiti et al.

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Fit with truncated Taylor series, then change $\mu^2 \rightarrow -\mu^2$

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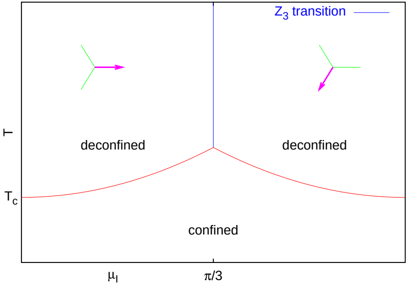

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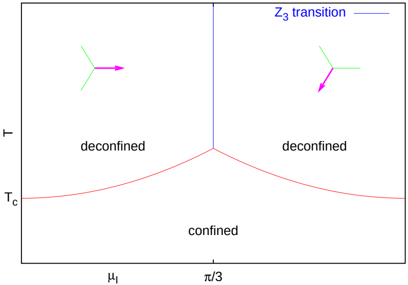

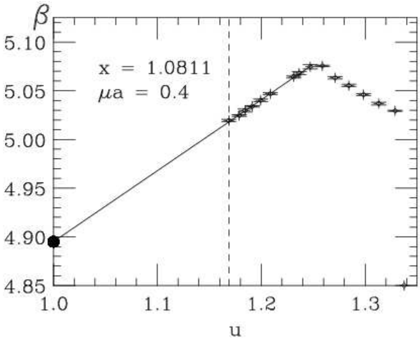

IIIa. Canonical ensemble

Hasenfratz & Toussaint; Alford et al.; Alexandru et al.; PdF & Kratochvila

IV. Different, sign pb.-free theories: isospin $\mu = \mu_u = -\mu_d$ Kogut & Sinclair

phases of determinants cancel: $Z = \int \mathcal{D}U \, |\det M(\mu)|^{N_f} e^{-S_g} \rightarrow$ "phase-quenched"

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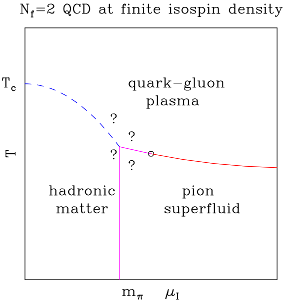

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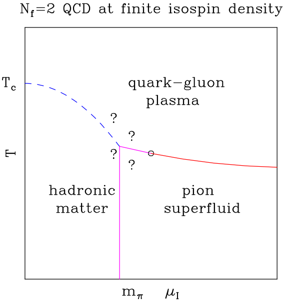
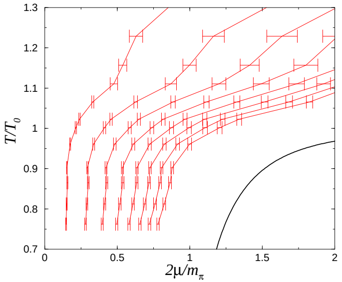
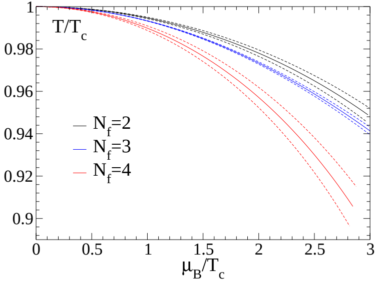


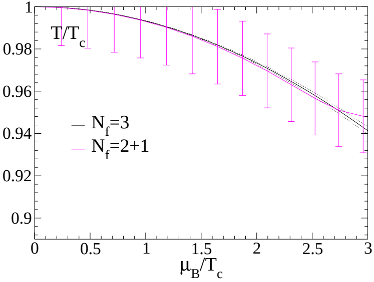


Sign problem severe (intractable?) when $\mu_q > m_\pi/2$ Splittorff

Transition temperature vs μ

All methods $\approx$ agree for $\frac{d^2 T_c}{d\mu^2}|_{\mu=0}$!





$\frac{T_c(\mu)}{T_c(\mu=0)} = 1 - \mathbf{c}(\frac{\mu}{\pi T})^2 + \ldots$

$c = 0.500(67), 0.602(9), 0.932(97)$ for $N_f = 2, 3, 4$ $\sim N_f$ Toublan

c insensitive to m_q if $m_q \ll \pi T$

Higher derivatives must bend the curve down ($\mu_c < 670$ MeV at $T = 0$)

On to the critical point

- Taylor expansion of pressure: $\frac{\Delta p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T)(\frac{\mu}{T})^2$

radius of convergence $\equiv$ distance to nearest singularity

$$\text{e.g.:} \quad \rho_n = \left|\frac{c_0}{c_{2n}}\right|^{1/2n}, \qquad r_n = \left|\frac{c_{2n}}{c_{2n+2}}\right|^{1/2}, \qquad \rho, r = \lim_{n\to\infty} \rho_n, r_n$$

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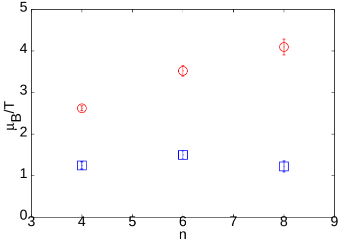

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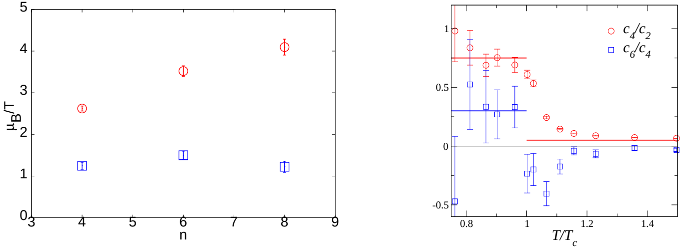

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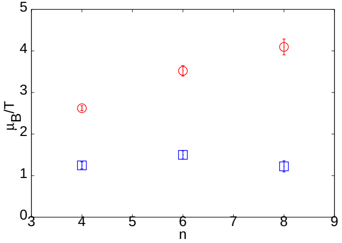

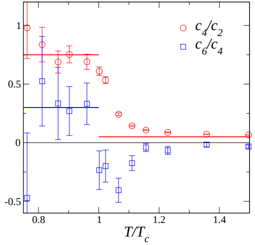

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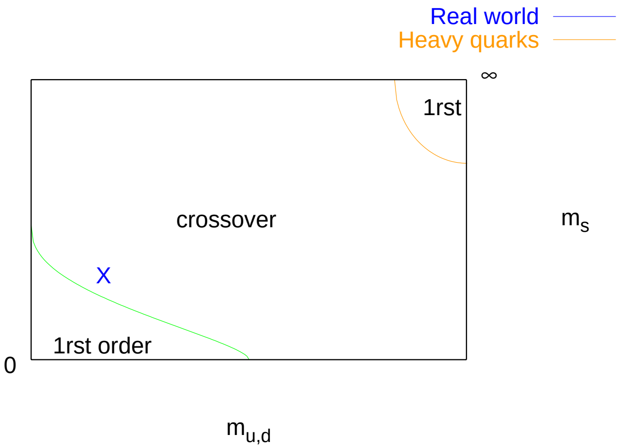
$N_f = 2$ interesting but [too] difficult: expect success only if μ_c “small”

Phase diagram vs $(m_{u,d}, m_s)$, T and μ

$\mu = 0$



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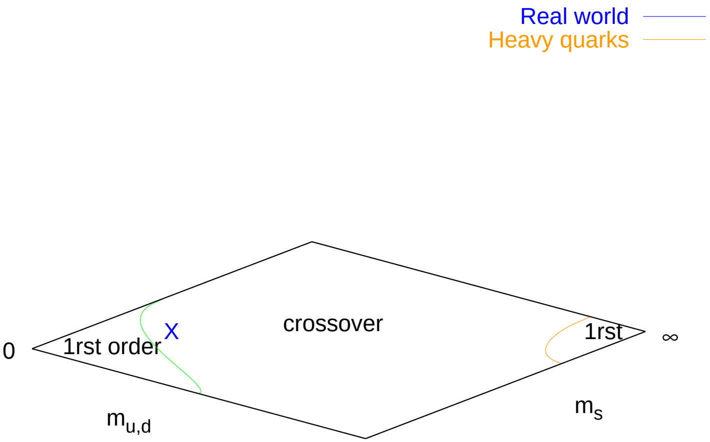

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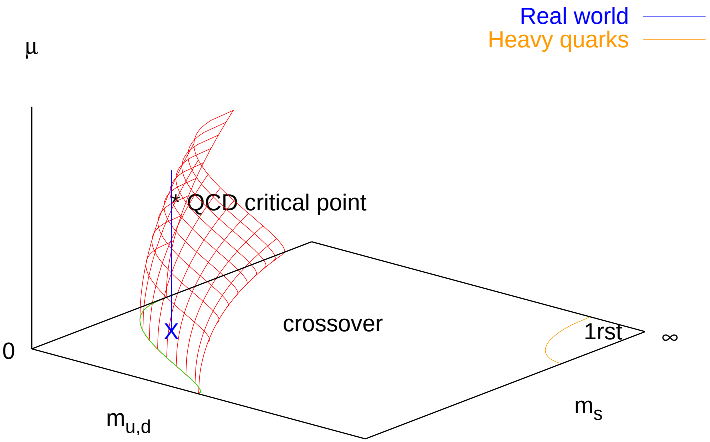

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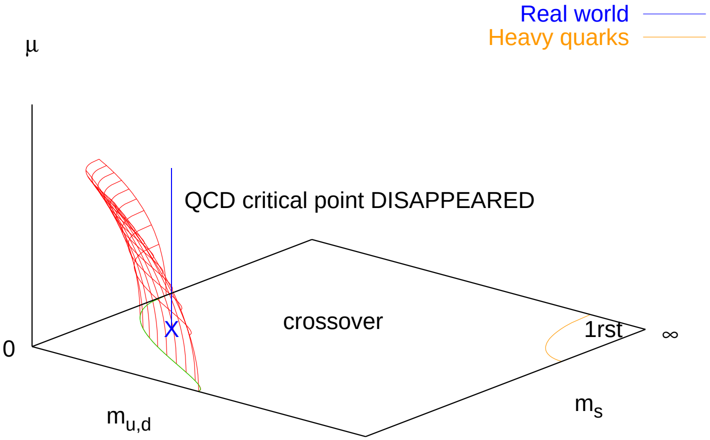

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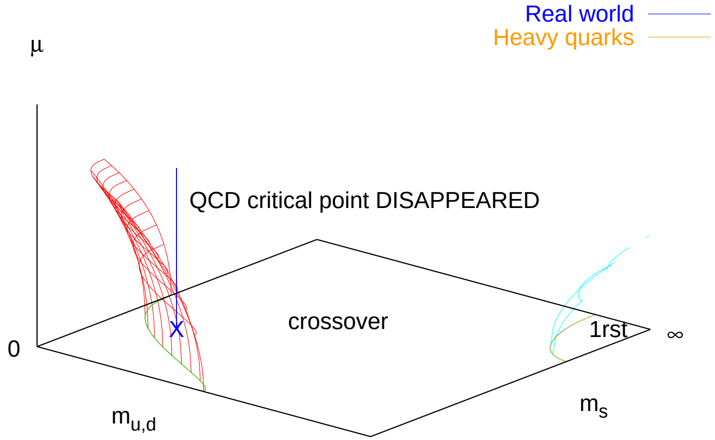
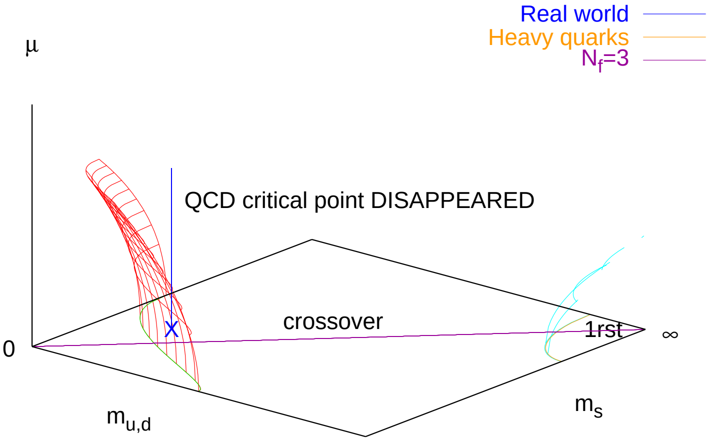
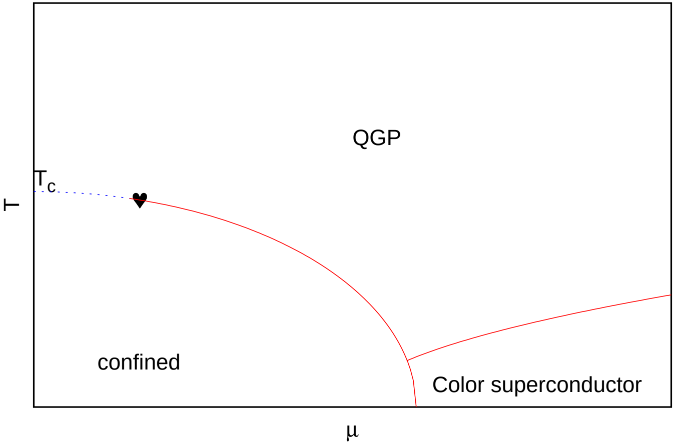


Phase diagram vs $(m_{u,d}, m_s)$, T and μ



(μ, T) phase diagram: vary m_q in $N_f = 3$ theory

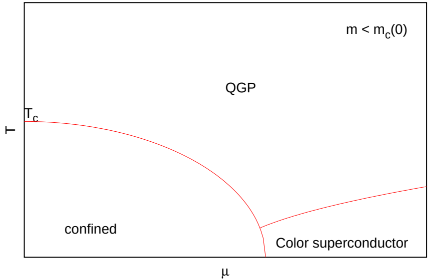
Conventional wisdom



How does the phase diagram change as m_q increases from 0 ?

(μ, T) phase diagram: vary m_q in $N_f = 3$ theory

Conventional wisdom



Normal

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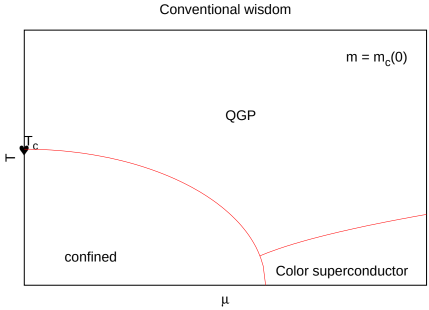

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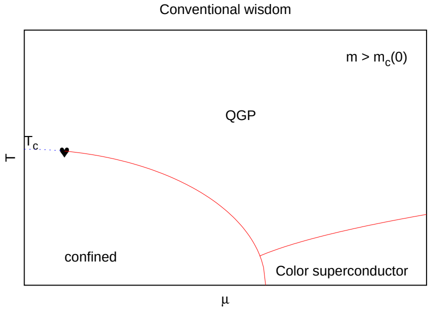

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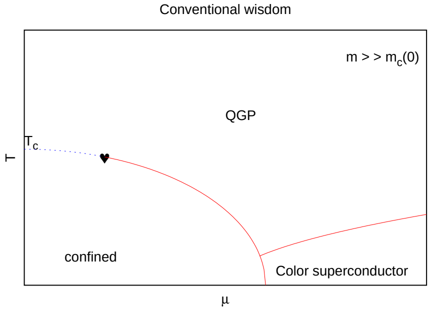

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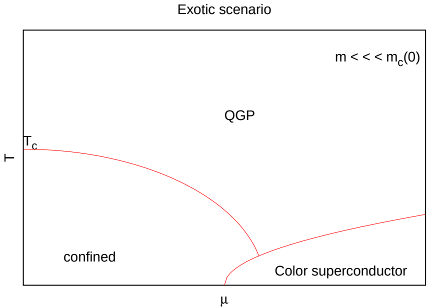
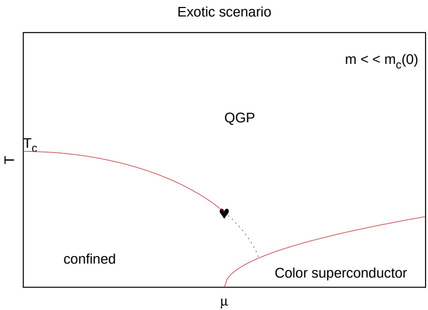


Exotic

(μ, T) phase diagram: vary m_q in $N_f = 3$ theory



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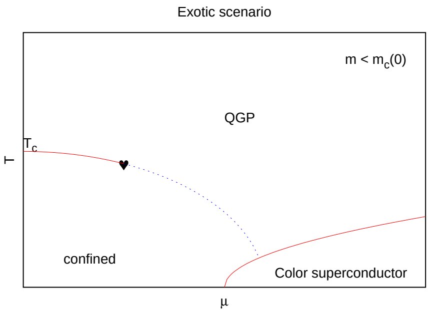

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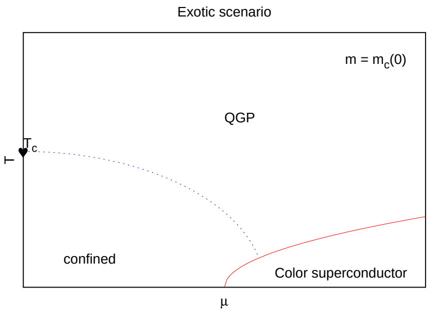

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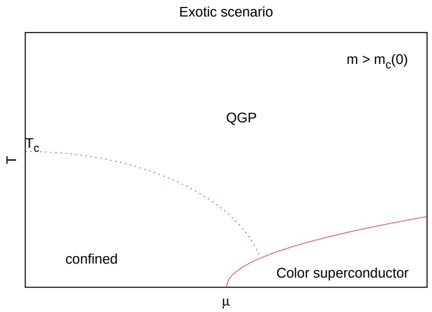

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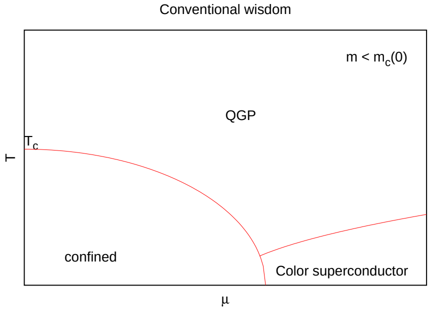

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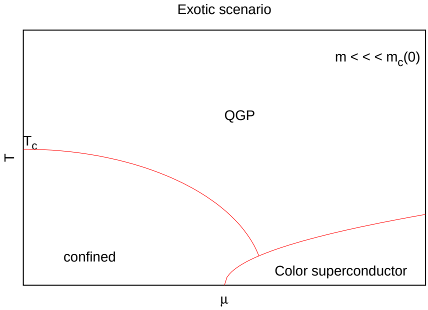

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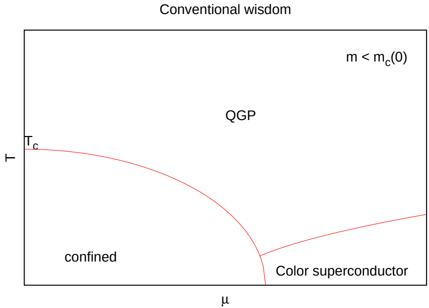

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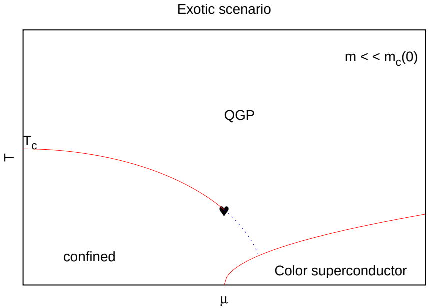

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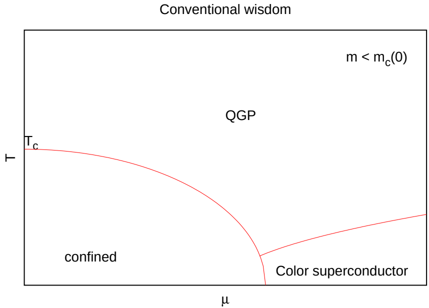

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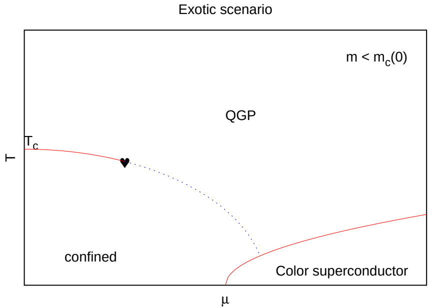

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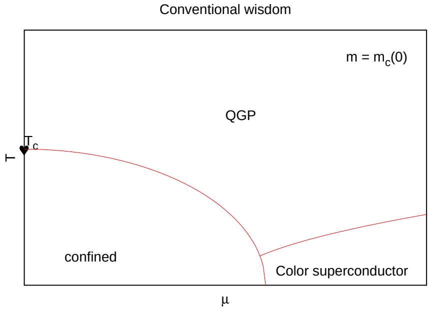

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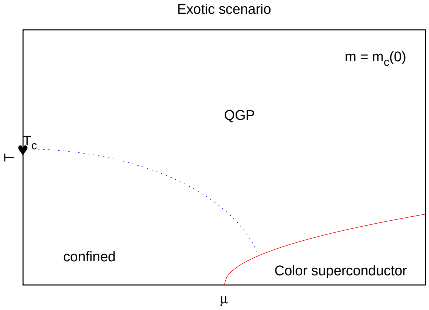

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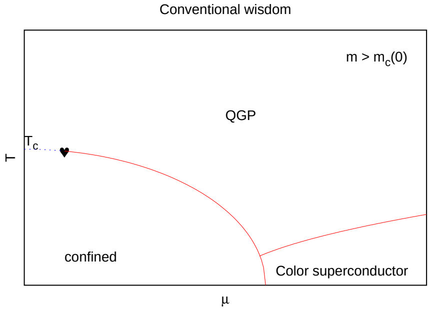

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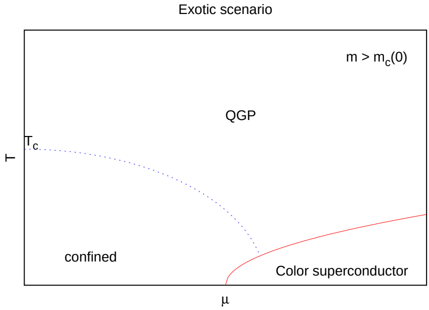

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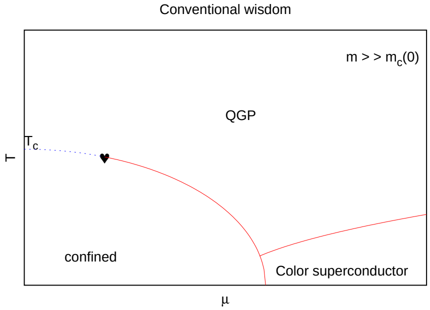

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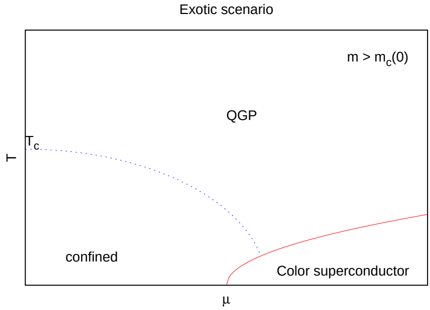

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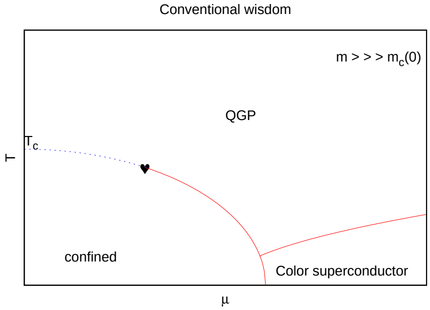

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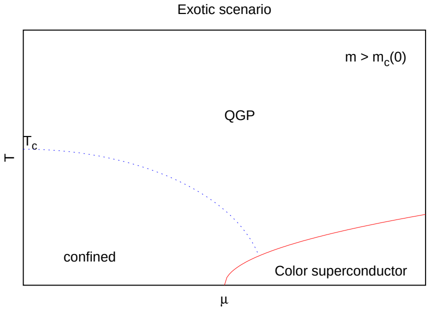
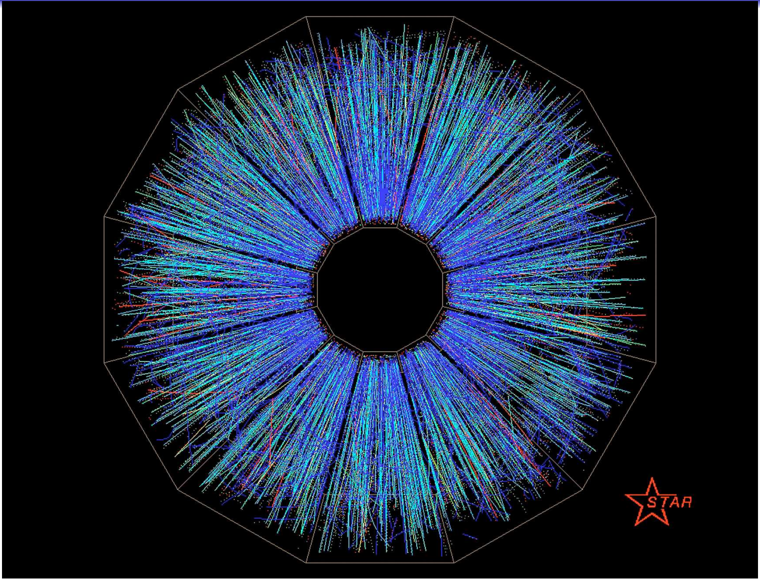


Exotic

Can there be no QCD critical point?

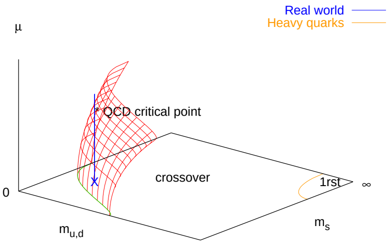


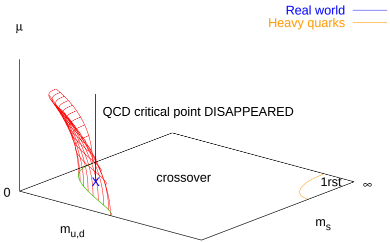
Can we make the experimentalists' life simpler ?

Can there be no QCD critical point?

- The answer is in the sign of $\frac{dm_c(\mu)}{d(\mu^2)}|_{\mu=0}$

Normal	Exotic





$\frac{dm_c(\mu)}{d(\mu^2)}|_{\mu=0} > 0$

1^{rst} order region expands

$\frac{dm_c(\mu)}{d(\mu^2)}|_{\mu=0} < 0$

1^{rst} order region shrinks

- First must tune quark mass to $m_c(\mu = 0)$.

Heavy quarks at finite density: Potts model

Static dense QCD Stamatescu; Blum, Hetrick, Toussaint

Heavy quarks: (M,μ)-dependent external field on Polyakov lines

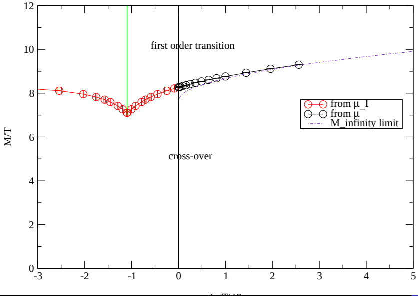
Simplify gauge action to $q = 3$ Potts:

$H = -\kappa \sum_{\langle ij \rangle} \delta(\sigma_i, \sigma_j) + h_+ \Phi + h_- \Phi^*$

$\sigma_i \in Z_3$; $\Phi = \sum_i \sigma_i$; $h_\pm = \exp(-\beta(M \pm \mu))$

$h = 0 \rightarrow 1^{rst}$-order PT, weakening when $h \neq 0 \rightarrow 2^{nd}$-order endpoint

Karsch & Stickan: $h_+ = h_-$ Alford et al.: $h_- = 0$

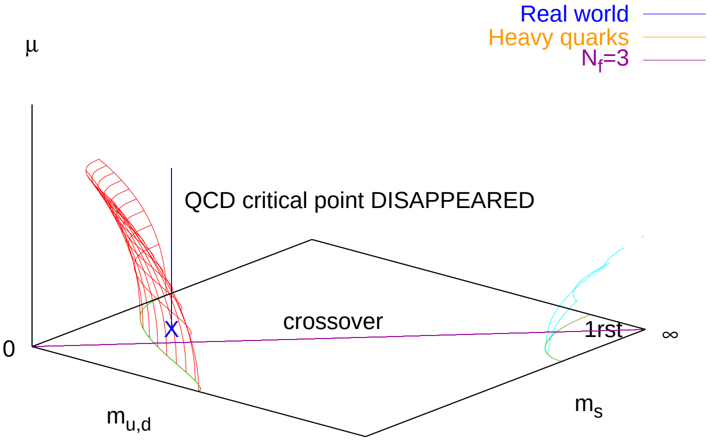


μ SHRINKS 1rst-order region

Analytic continuation ok

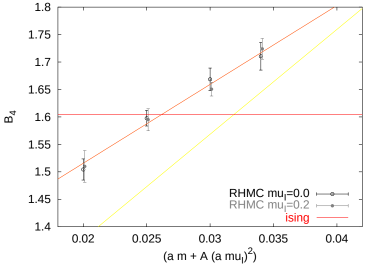
Kim et al., LAT05

Light quarks, $N_f = 3$



m_c versus μ_I

Enabling technology: RHMC versus R-algorithm



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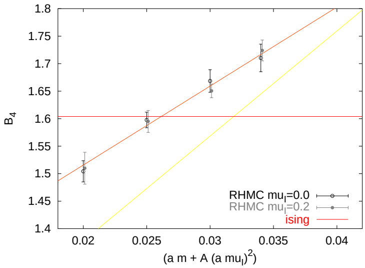

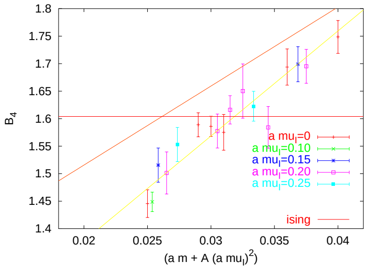

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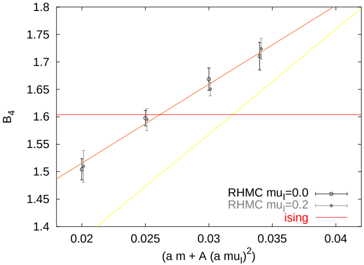
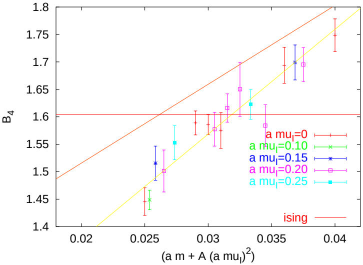




20% shift in $(am_c)(\mu = 0)$!

$$\frac{(m_c/T_c)(\mu)}{(m_c/T_c)(\mu{=}0)} = 1 + 0.07(30)\left(\frac{\mu}{\pi T}\right)^2 \rightarrow \frac{m_c(\mu)}{m_c(\mu{=}0)} = 1 - 0.53(31)\left(\frac{\mu}{\pi T}\right)^2$$

μ SHRINKS 1^{rst} order region again?

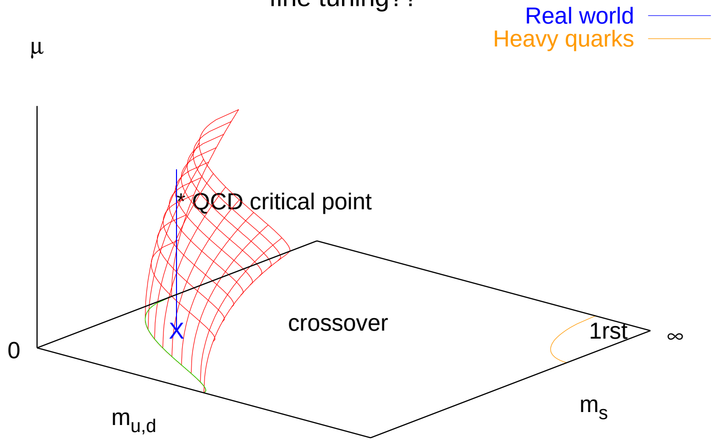
No critical endpoint?

$\frac{m_c(\mu)}{m_c(\mu=0)} = 1 - 0.53(31)\left(\frac{\mu}{\pi T}\right)^2$

less than 2 σ from zero; coarse lattice $a \sim 0.3$ fm

If critical point (T_c, μ_c), μ_c **extremely sensitive to quark masses**

fine tuning??



Canonical formalism on the lattice

$$Z_C(B) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\left(\frac{\mu_I}{T}\right)\, e^{-i3B\frac{\mu_I}{T}} Z_{GC}(\mu = i\mu_I)$$

The dependency on μ_I is in $\det M(U, i\mu_I)$ only!

Strategy: sample $Z_{GC}(i\mu_I)$ at some fixed $\mu_I = \mu_{I_0}$

$$\frac{Z_C(B)}{Z_{GC}(i\mu_{I_0})} = \frac{1}{Z_{GC}(i\mu_{I_0})}\int dU e^{-S_g[U]} \det(U, i\mu_{I_0}) \times \frac{1}{\det(U, i\mu_{I_0})}\frac{1}{2\pi}\int_{-\pi}^{\pi} d\left(\frac{\mu_I}{T}\right)\, e^{-i3B\frac{\mu_I}{T}} \det(U, i\mu_I)$$

$$= \left\langle \frac{1}{\det(U, i\mu_{I_0})}\frac{1}{2\pi}\int_{-\pi}^{\pi} d\left(\frac{\mu_I}{T}\right)\, e^{-i3B\frac{\mu_I}{T}} \det(U, i\mu_I)\right\rangle_{\beta, i\mu_{I_0}}$$

Fourier transform each determinant $\longrightarrow$ work $\sim L_s^9 \times L_t$

Hasenfratz & Toussaint

From canonical to grand canonical

Version 1: Fugacity Expansion $(\rho = \frac{B}{V})$

$$\langle B(\mu)\rangle = \frac{\sum_{B=-V}^{V} B\, Z_C(B) e^{3B\frac{\mu}{T}}}{\sum_{B=-V}^{V} Z_C(B) e^{3B\frac{\mu}{T}}}$$

Version 2: Saddle Point Approximation

$$Z_{GC}(\mu) = \int d\rho\, e^{-\frac{V}{T}(f(\rho)-3\mu\rho)}$$

$$\rightarrow \quad \mu(\rho) = \frac{1}{3}f'(\rho) \underset{V<\infty}{\approx} \frac{V}{3}\left(f(\rho)-f(\rho-1/V)\right)$$

$$\frac{\mu(B)}{T} = \frac{F(B)-F(B-1)}{3T}$$

Setup: 6^3x4, $a \sim 0.3$ fm, $N_f = 4$ KS fermions, $m_\pi \sim 350$ MeV

$\Rightarrow$ 1rst-order transition expected for all μ

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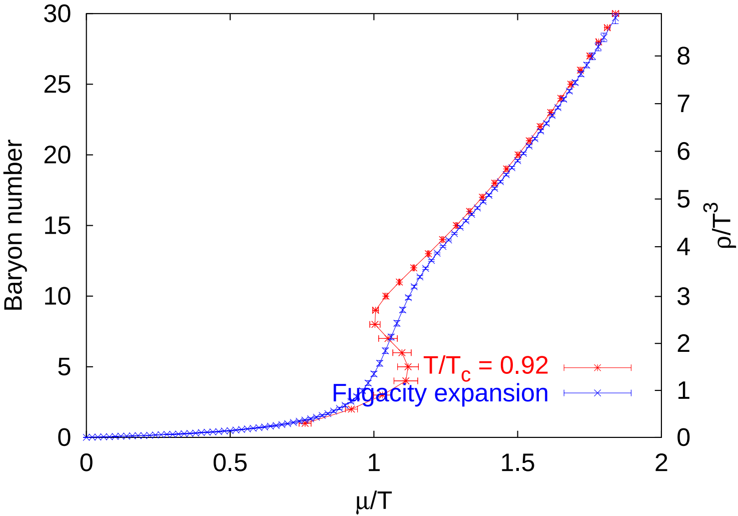
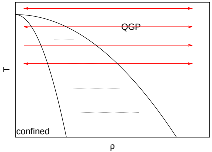
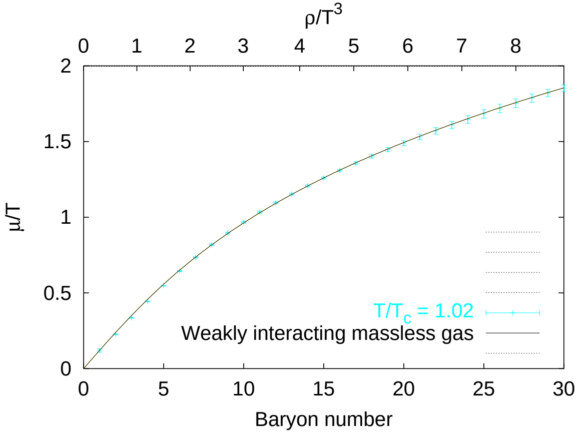


Baryon number
ρ/T^3
μ/T
T/T_c = 0.92
Fugacity expansion
30
25
20
15
10
5
0
8
7
6
5
4
3
2
1
0
0.5
1
1.5
2

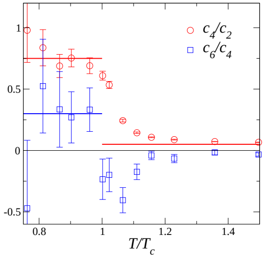
$\frac{\mu(B)}{T} = \frac{F(B)-F(B-1)}{3T}$



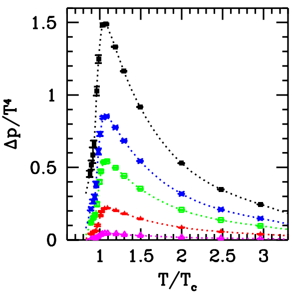


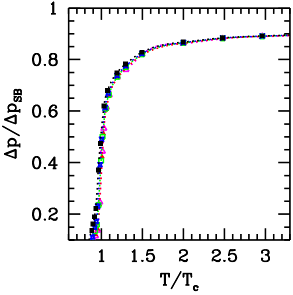
$$\frac{\rho(\mu)}{T^3} \approx 2b_2c_2\left(\frac{\mu}{T}\right) + 4b_4c_4\left(\frac{\mu}{T}\right)^3 \to b_2 = 0.92(1), b_4 = 2.18(1)$$

Quarks interact weakly in the QGP ?



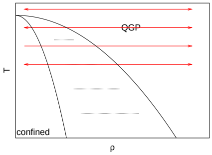
Bielefeld-Swansea

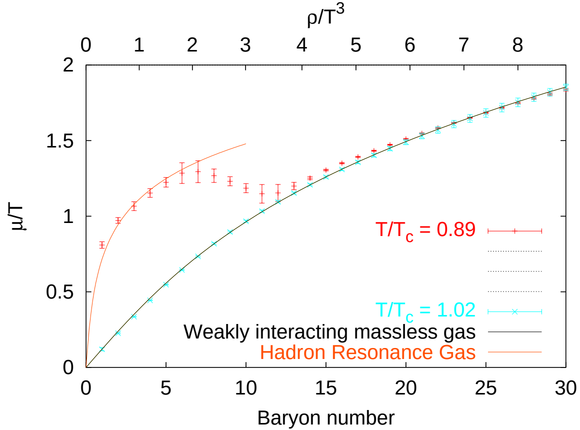




Csikor et al.

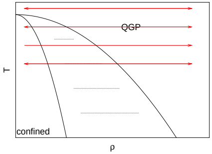
Low density phase consistent with Hadron Resonance Gas

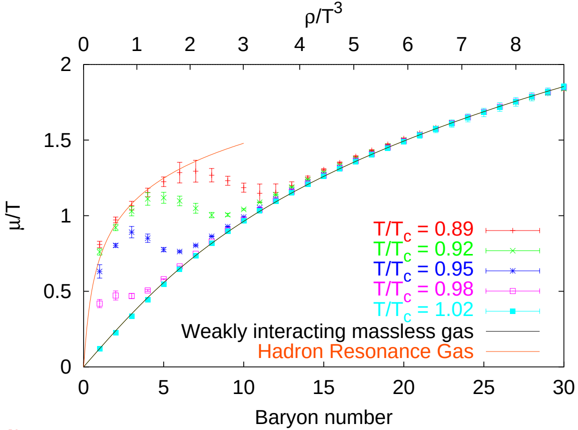




$$\frac{\rho(\mu)}{T^3} = 3F(T)\sinh\frac{3\mu}{T} \to F(T) = 0.070(6)$$

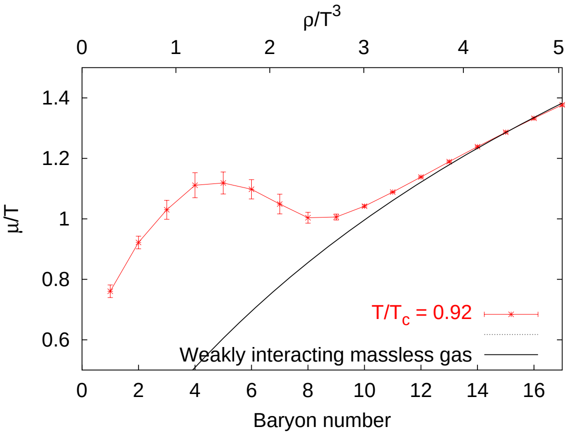
Low density phase consistent with Hadron Resonance Gas





Good accuracy up to $\frac{\mu}{T} \sim 2$; fluctuations in transition region physical

Maxwell Construction



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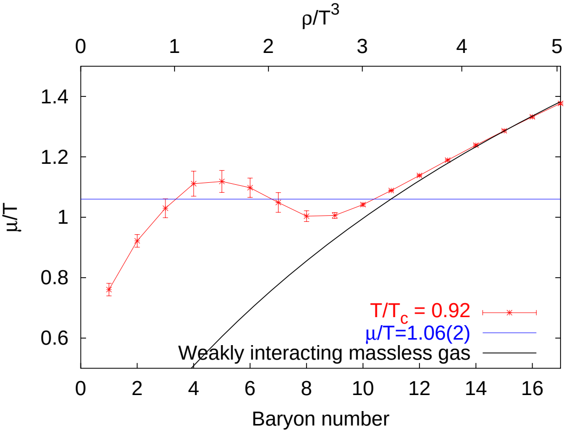
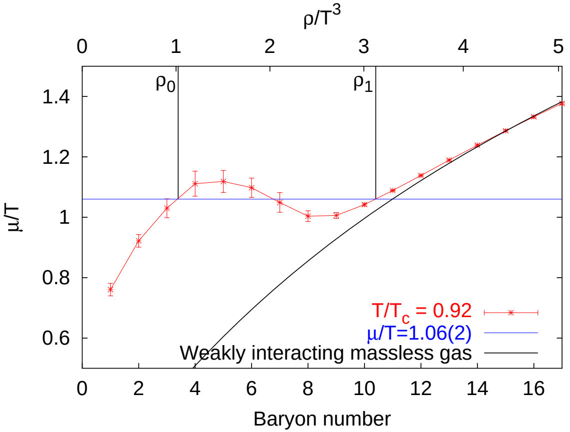


$\frac{1}{T}\int_{\rho_0}^{\rho_1} d\rho (f'(\rho) - \mu) = 0 \longrightarrow f(\rho_0) = f(\rho_1)$, ie. phase transition

Maxwell Construction



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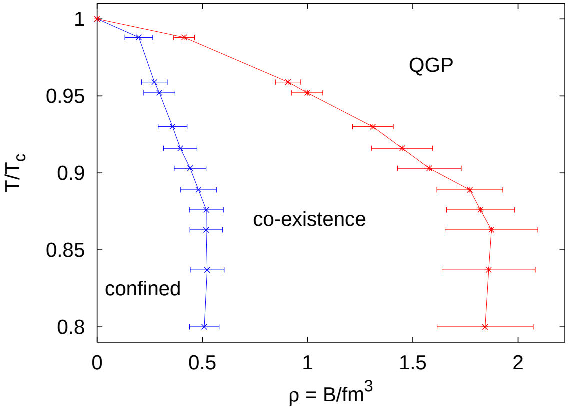
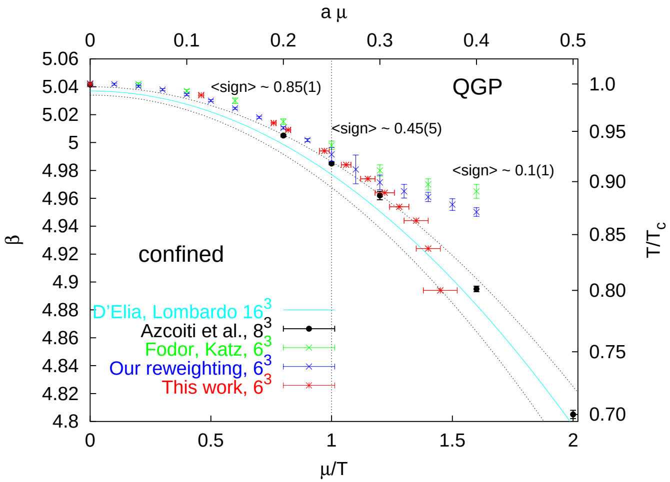
Phase Diagram $T - \rho$



Compare ρ_0 with nuclear density $0.17/\mathrm{fm}^3$

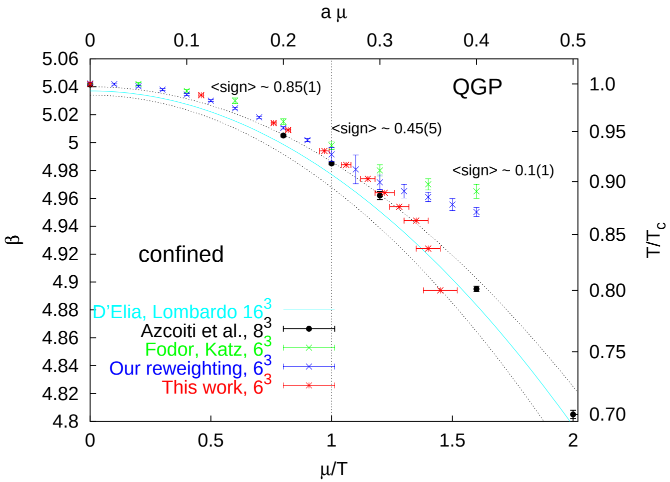
QGP with free massless gas Ansatz: $\rightarrow \rho_0(T \sim 0) \sim \frac{1}{4}\rho_1(T \sim 0)$

Phase Diagram $T - \mu$: comparing apples with apples



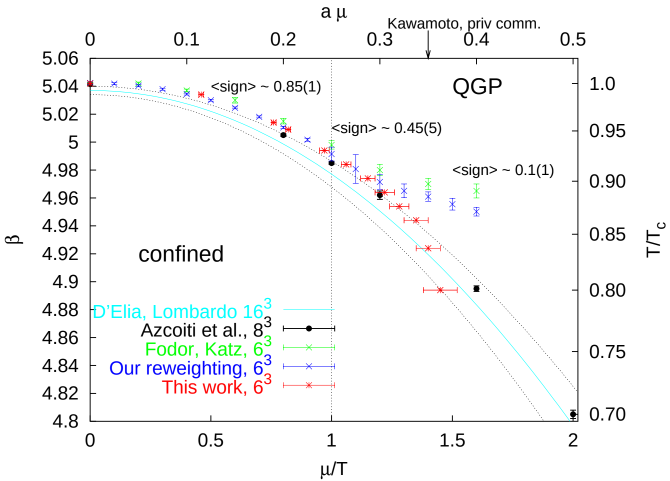
i) reweighting becomes unreliable

Phase Diagram $T - \mu$: comparing apples with apples



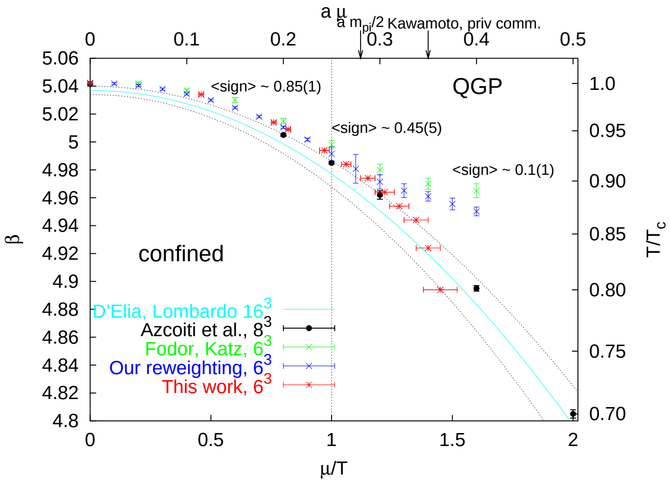
ii) systematic error of analytic continuation not studied at $\frac{\mu}{T} > 1$

Phase Diagram $T - \mu$: comparing apples with apples



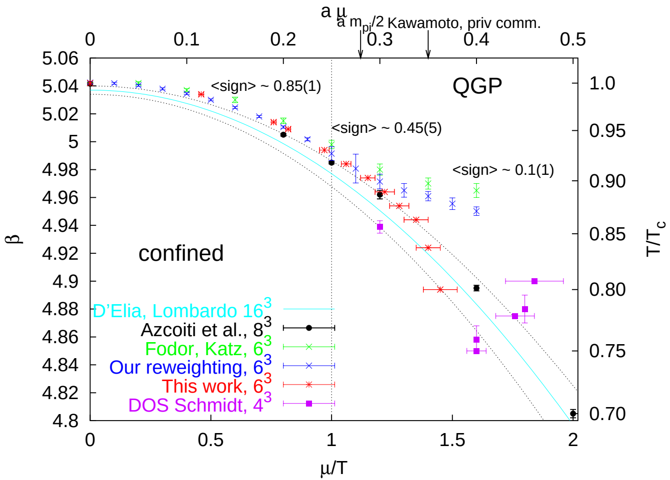
iii) $\beta_c(a\,\mu)$ must bend down to match expectations at $\beta = 0$

Phase Diagram $T - \mu$: comparing apples with apples



iv) Trouble starts when $\mu > m_\pi/2$ Splittorff

Phase Diagram $T - \mu$: comparing apples with apples



v) DOS results? See next talk Schmidt

Conclusions

- Sign & overlap problems at finite μ not for the timid
- Time has come to assess systematic errors
- $\frac{d^2 T_c}{d\mu^2}\Big|_{\mu=0}$ under control (continuum limit?)
- Critical point unlikely to be at small μ (fine tuning?)
- $N_f \neq 2$ very interesting: - critical point movable to $\mu = 0$
  - nuclear matter more stable?
- Canonical formalism