Bulk thermodynamics of $SU(N)$ lattice gauge theories at large-N

Barak Bringoltz

University of Oxford

Based on: BB and Michael Teper, hep-lat/0506034

I. Outline of the talk

II. Sketch the motivation - what is the pressure deficit at large-N ? .

III. Describe the integral method (briefly).

IV. "Reduction at large-N", or are these volumes large enough ? .

V. Results: method of presentation

VI. Results for p, and Δ.

VII. Summary and implications.

II. Motivation:

What replaces Hadronic phase after deconfinement ?

Recent years : evidences that for $T_c < T \lesssim 2T_c$, simple QGP is unsuitable:

- RHIC : elliptic flow and low viscosity.
- Lattice:
  - $10 - 20\%$ deviation from free gas up to $\sim 4T_c$. Boyd et al. '96
  - Small viscosity in pure $SU(3)$ Nakamura et al. '98, '04.
  - Survival of heavy quarkonia states up to $\sim 1.5 - 1.7T_c$. Petreczky '04 (Lattice04 review)

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Inspired many theoretical approaches, to mention only a few:

1. Quasi-gluons and quarks with $m_{q,g}(T)$, Peshier et al. '96, Levai and Heinz '97.
2. Perturbation up to $\mathcal{O}(g^6)$ + 3d Euclidean theory Kajantie et al. '02.
3. Resummations Blaizot et al. '03 (review).
4. Loosely bound states - relates three phenomena, Shuryak and Zahed '04.

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large–N numerical study can test/constrain these and point to their important ingredients.

The study of large-N is too vast to review here .

1. Loop models Daamgard and Patkos '86,Pisarksi '00, Dumitru et al. '04 ,Eguchi-Kawai inspired models Billo et al. '94, Small volumes Aharony et al. '03,'05.

   → these become soluble at large–$\boldsymbol{N}$.

2. $\mathcal{N} = 4$ SUSY with large–$\boldsymbol{N}$, and $\boldsymbol{g^2 N \gg 1}$, where behaviour is reminiscent of QCD's Gusber et al. '98, Policastro et al. '01.

3. Have accurate numerical results of deconfinement in $4d$ pure gauge with $N \leq 8$ by Teper and Lucini '02, Del Debbio et al. '04, Lucini, Teper and Wenger '05, BB and Teper, '05, Bursa and Teper '05 $[\boldsymbol{T_c, \xi(T_c), L_h, \ldots}]$.

   → Good starting point for an accurate calculations.

⇓

We numerically calculate bulk thermodynamics of pure lattice gauge theories for $N > 3$.

III. The integral method Boyd et al '96

Want to evaluate the free energy $F(T,V) = -T \log Z$

When $V = \infty$, then $F = V f(T)$, and $p(T) = \frac{T}{V} \log Z = \frac{1}{L_s^3 L_t a^4} \log Z$.

Useful to define $\Delta/T^4 = \frac{\partial(p/T^4)}{\partial \log T} \Rightarrow \begin{cases} \epsilon = \Delta + 3p \\ s\,T = \Delta + 4p \end{cases}$

On the lattice: express p, Δ in terms of MC averages.

1. "Differential" but larger $\boldsymbol{N}$ $\rightarrow$ smaller $\boldsymbol{L_t}$
   $\rightarrow \boldsymbol{a^{-1} \simeq 1.3}$GeV near $\boldsymbol{T_c}$ $\rightarrow$ can get $p(T_c) < 0$ Svetitsky and Fucito, '83

2. Direct evaluation of density of states $\rightarrow$ modern methods Wang and Landau '01.
   But, preliminary checks did not converge.

3. "Integral method" Boyd et al. '96 (for pure $SU(3)$), Engels et al. '90.

We choose the integral method :

$$p(T) = \frac{1}{a^4(\beta)L_t L_s^3}\log Z = \frac{1}{a^4(\beta)L_t L_s^3}\int_{\beta_0}^{\beta}\underbrace{\frac{\partial \log Z}{\partial \beta'}}_{\substack{\downarrow \\ 6L_s^3 L_t u_p(\beta')}} d\beta' = \frac{6}{a^4}\int_{\beta_0}^{\beta} u_p(\beta')d\beta'$$

Conventional regularization : $p(T;\beta) \to p(T;\beta) - p(0;\beta) \Rightarrow p(T=0) \equiv 0$.

$$u_p \to \delta u_p \equiv u_p(L_s^3 L_t) - u_p(L_s^4)$$

$$p/T^4 = 6L_t^4 \int_{\beta_0}^{\beta} \delta u_p(\beta')\, d\beta' \quad \left[+p/T^4\right]_{T_0}; \qquad \Delta/T^4 = 6\,\delta u_p(\beta) \times \frac{\partial \beta}{\partial \log a^{-1}}.$$

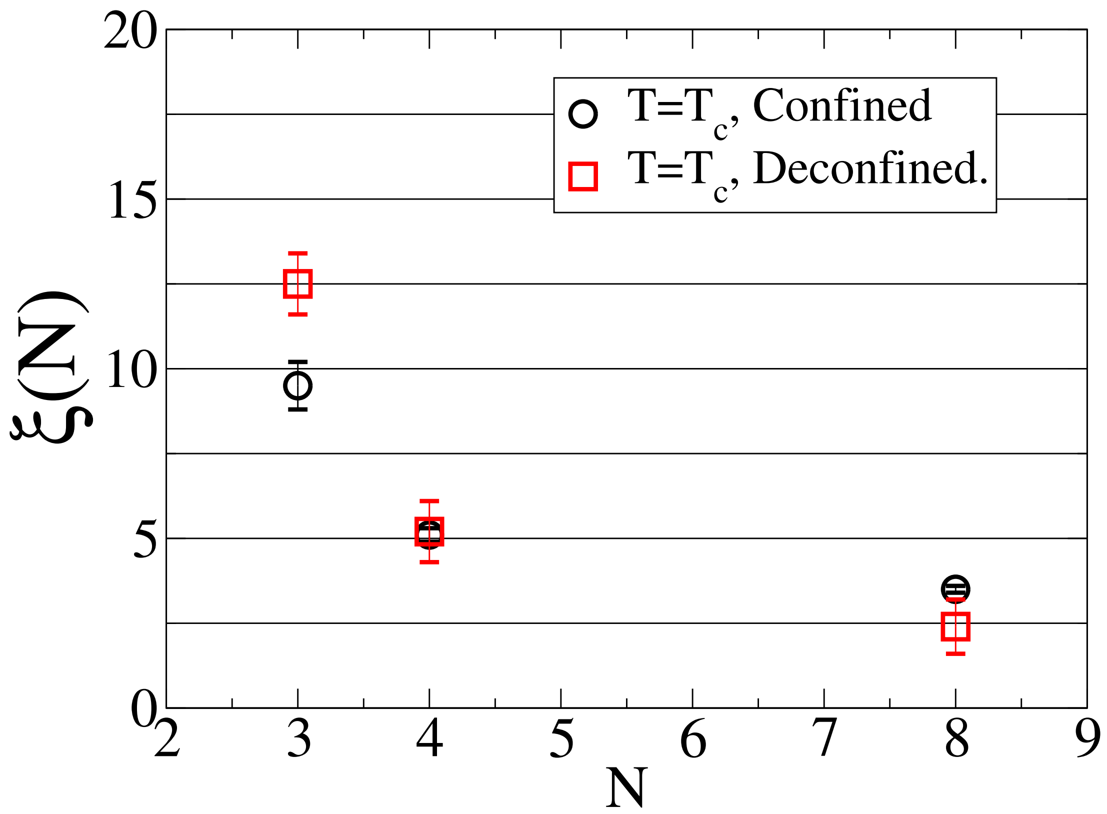
To conclude: Need 2 MC sets on an L_s^4, and an $L_s^3 L_t$ for each $\beta \in [\beta_0, \beta_{\max}]$. Also choose $\beta_0 < \beta_c$ (then $\delta u_p(\beta_0) \simeq 0 \to \left(p/T^4\right)_{T_0} \simeq 0$) .

MC's info : Pure gauge with $\boldsymbol{L_t = 5}$: $\boldsymbol{SU(4)}$ on $\boldsymbol{16^3 5}$, $\boldsymbol{SU(8)}$ on $\boldsymbol{8^3 5}$, Compare with $\boldsymbol{SU(3)}$ of Boyd et al. '96, where $\boldsymbol{L_t = 4, 6, 8}$, $\to$ supplement for $\boldsymbol{SU(3)}$ on $\boldsymbol{20^3 5}$.

Reduction at large-N or are these volumes large enough ?

Reduction : Eguchi and Kawai '82, recently- Neuberger and Narayanan '03 suggests fast approach to $V = \infty$ at large-N.

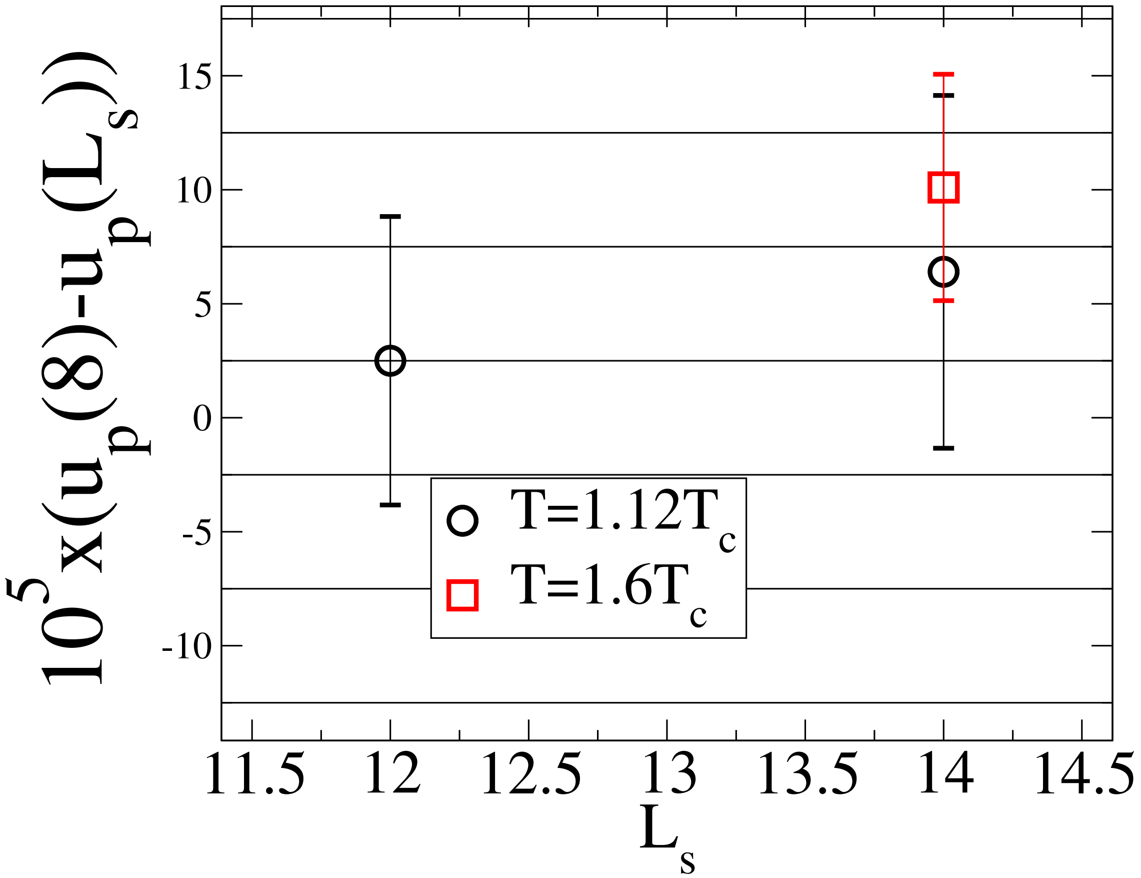
- Indeed: ξ decreases with N (and decreases from T_c) Lucini et al. '05.



Reduction at large-N or are these volumes large enough ?

Asymmetric lattices, $T \neq 0$: our choices of L_s are **OK** when studying $u_p(L_s)$.

At most 2σ variations.

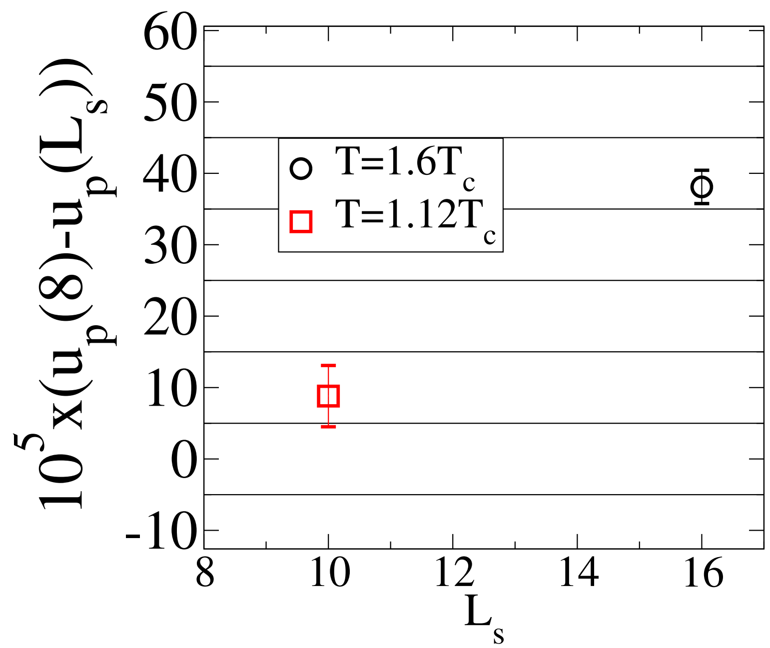


Reduction at large-N or are these volumes large enough ?

Symmetric lattices, $T = 0$: need $\underline{V^{1/3} \gg 1/T_c}$, since at $N \gg 1$ have phase transitions there e.g. Neuberger and Narayanan '03:

- For $N = 8$: $1.6 \geq V^{1/3} T_c \geq 1.0$. ✗

Investigating $u_p(L_s)$ we find **a huge 16σ deviation for $SU(8)$**.



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- For $N = 8$: $1.6 \geq V^{1/3} T_c \geq 1.0$. ✗
- **For $N = 8$, 8^4 is too small** → We use (Lucini et al. '05)'s $u_p(L_s)$ from $L_s \leq 16$, to fit $u_p^0(\beta)$:

$$u_p(\beta) = u_p^{PT}(\beta) + \frac{\pi^2}{12}\frac{G_2}{N} a^4(\beta) + c_4 g^8 + c_5 g^{10}$$

$$\hookrightarrow \mathcal{O}(g^6)$$ Alles et al. '98

Finally: separate phases with $\beta_c(V = \infty)$ Lucini et al. '05:

→ Only when $T \simeq T_c$ for $SU(4)$, and $SU(8)$.

→ For $N = 3$ the volume is too small to separate.

Another aspect of reduction at large-N

Reduction a lá Eguchi-Kawai: at $N = \infty$

- Nothing depends on T if $T < T_c$ Gocksch and Neri ‘84.
- Nothing depends on V if $V > V_c$ at $T = 0$ Recent Neuberger et al. ‘03-‘04.

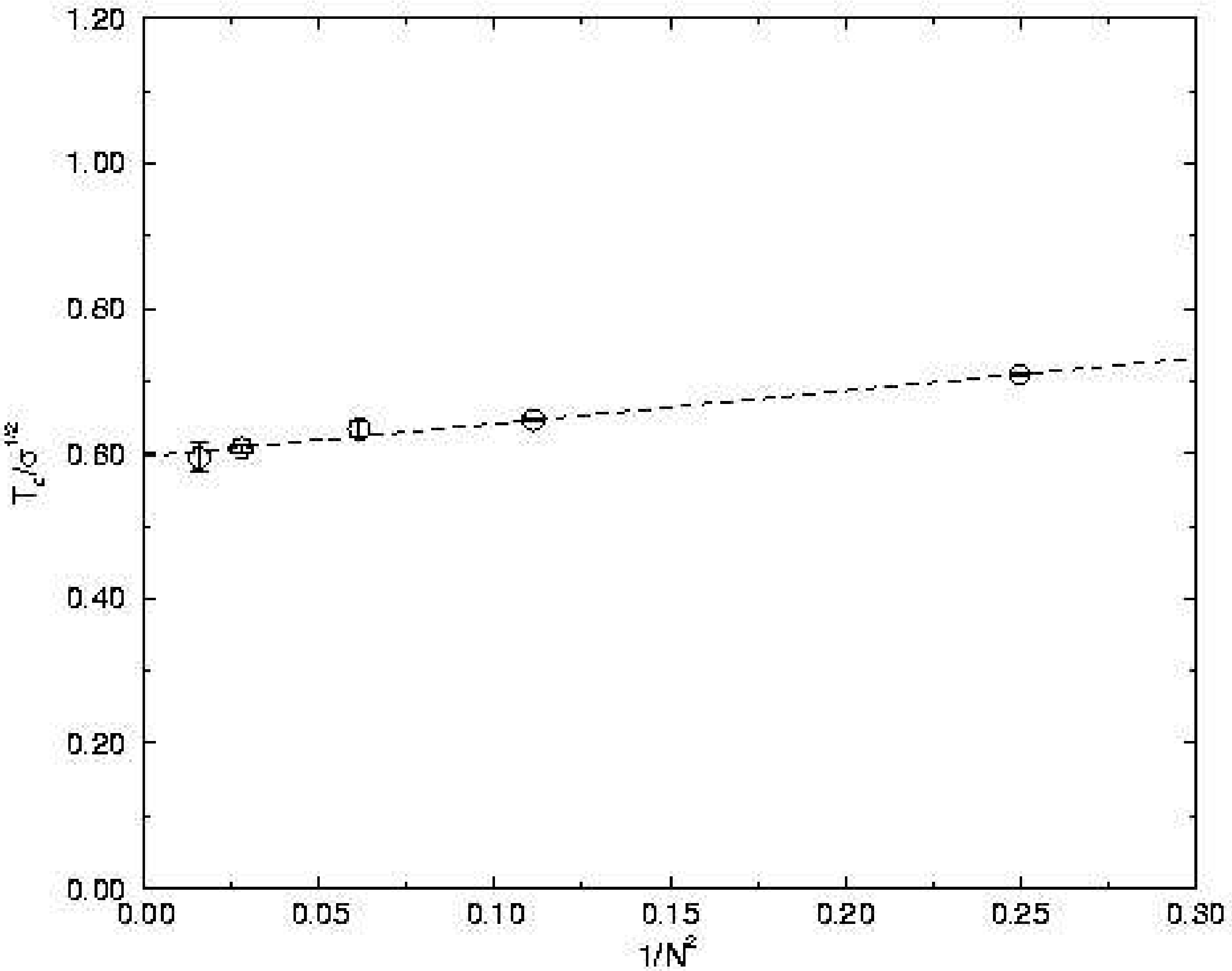
Is $u_p(T_c^-) = u_p(0)$? :

- $SU(3)$: $\mathbf{15\sigma}$ $\times$. data from Boyd et al. ‘96
- $SU(4)$: $\mathbf{1.7\sigma}$ $\surd$.
- $SU(8)$: $\mathbf{2.1\sigma}$ $\surd$.

Which means that $\left[p/T^4\right]_{T<T_c} = 0$ for larger $N \rightarrow$ <u>very small</u> systematic error from integration for larger N.

V. The physical scale, what is $T/T_c(\beta) = ?$

The value of $T_c/\sqrt{\sigma}$ Lucini et al. '03,'05 $(a \to 0, V \to \infty)$



The most natural scale here is T_c.

$\longrightarrow$ this requires $\beta_c(L_t, L_s) \longrightarrow$ a very large-scale project (need many L_t, L_s).

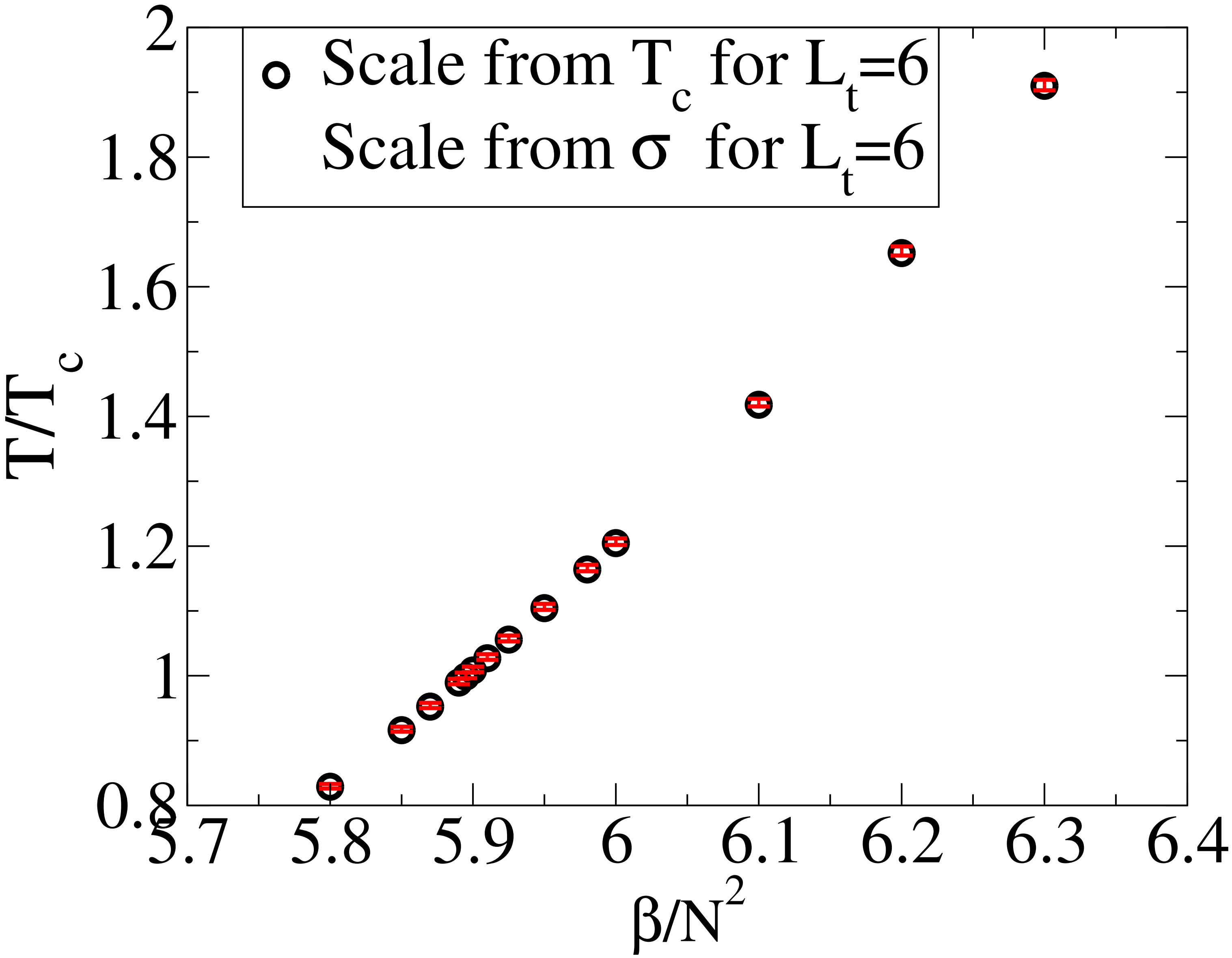
One can also fix the scale with $\sqrt{\sigma}$ using the $(a\sqrt{\sigma})_\beta$ interpolation of Lucini et al. '05

Possible differences are due to $\mathcal{O}(a^2)$ corrections to $\frac{T_c}{\sqrt{\sigma}}$, and

- For $SU(3), SU(8)$: $\frac{T_c}{\sqrt{\sigma}}$ at $a = 1/5T_c, 1/8T_c$ are the same with errors.
- For $SU(4)$: there is a 5σ difference between $a = 1/5T_c, 1/8T_c$, which is however only $\sim 2\%$ $\longrightarrow$ a small overestimate of T/T_c when $T/T_c \simeq 8/5$.

$\longrightarrow$ We fix the scale with $\sqrt{\sigma}$

As a consistency check : compare $T/T_c(\beta)$ from Boyd et al. '96 made with T_c for $SU(3)$, and


Scale from T_c for L_t=6
Scale from σ for L_t=6
T/T_c
β/N^2
2
1.8
1.6
1.4
1.2
1
0.8
5.7
5.8
5.9
6
6.1
6.2
6.3
6.4

VI. Results

We work at fix lattice spacing $a \simeq 1/5T$:

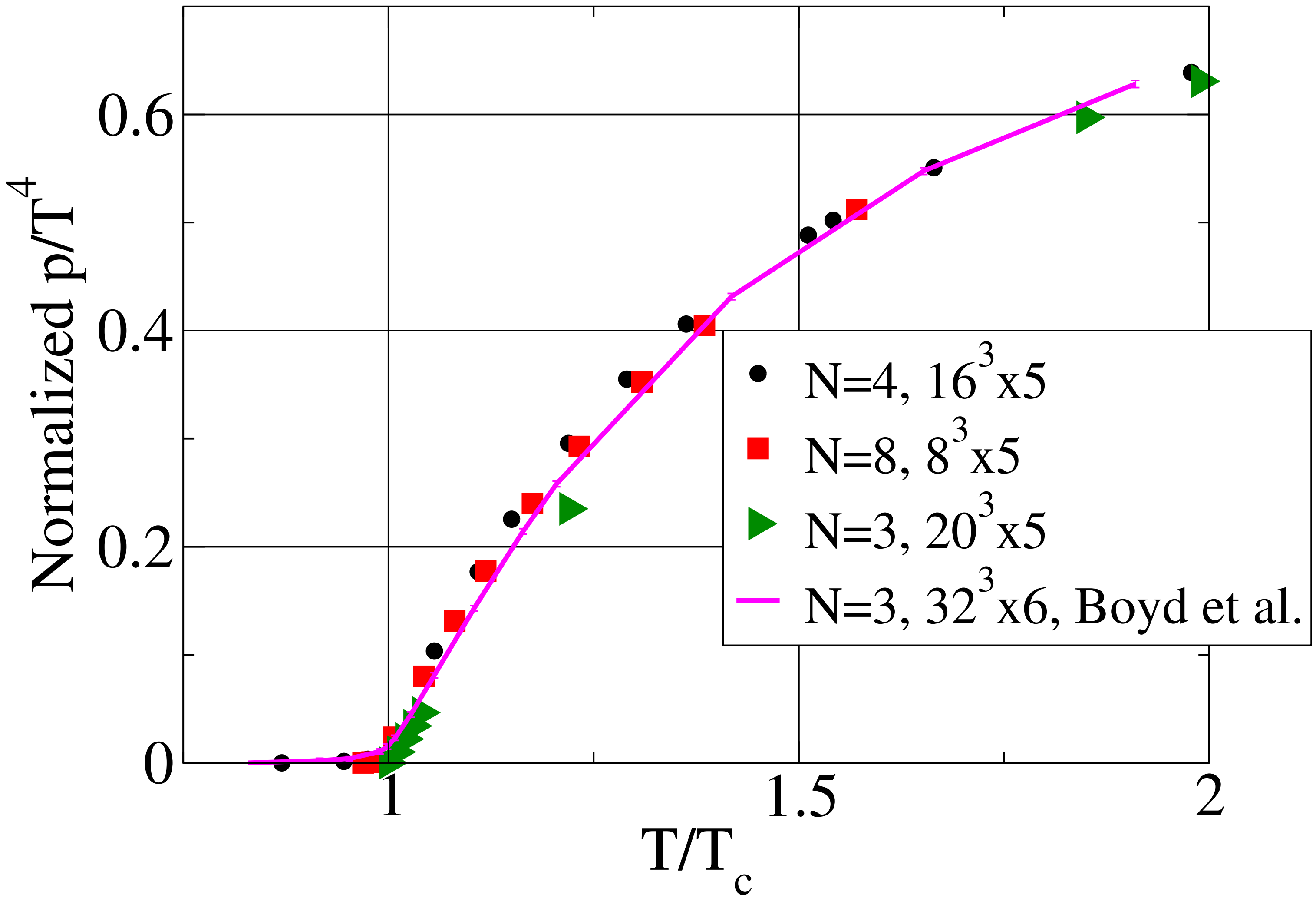
- Have $\mathcal{O}(1/L_t^2)$ to the free-gas Engels et al. '99. If $L_s \to \infty$, then

$$p_{\rm free}/T^4 \quad = \quad (N^2-1)\frac{\pi^2}{45}\left[1+\underbrace{\frac{8}{21}\left(\frac{\pi}{L_t}\right)^2+\frac{5}{21}\left(\frac{\pi}{L_t}\right)^4}_{\sim 19\% \ \text{for} \ L_t=5.}+\mathcal{O}(L_t^{-6})\right].$$

We normalize to the lattice free gas, to get $p/T^4 \to 1$, $\epsilon/T^4 \to 3$, $s/T^3 \to 4$, and therefore we normalize Δ/T^4 accordingly.

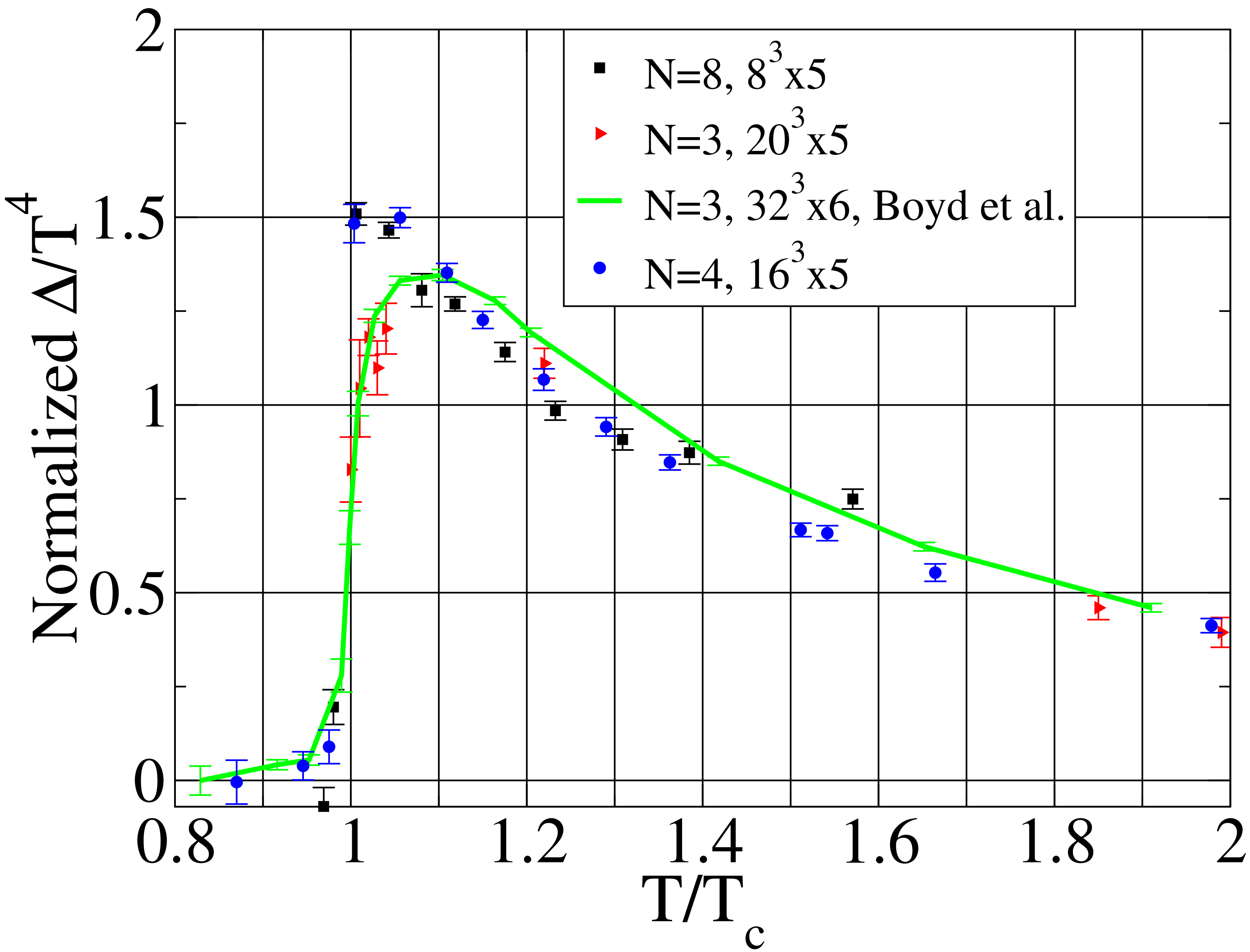
VII.a. Results-Normalized pressure (symbols' size=errors)

Pressure plots lie almost on top of each other.



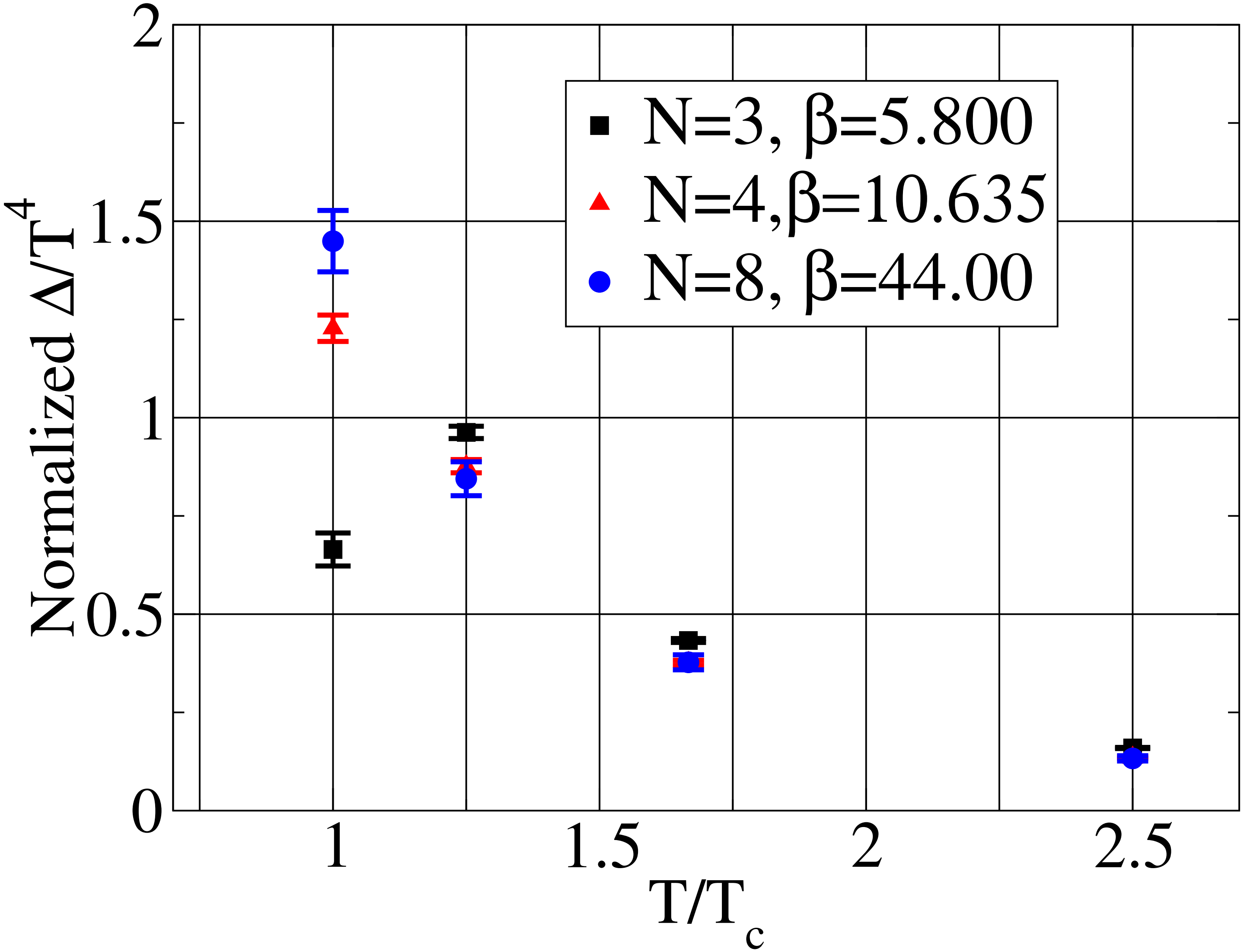
VII.b. Results-Normalized Δ

(1) Δ has modest $O(1/N)$ corrections.
(2) At $T = T_c$ Δ is different, possibly due a weak 1st order in $SU(3)$.



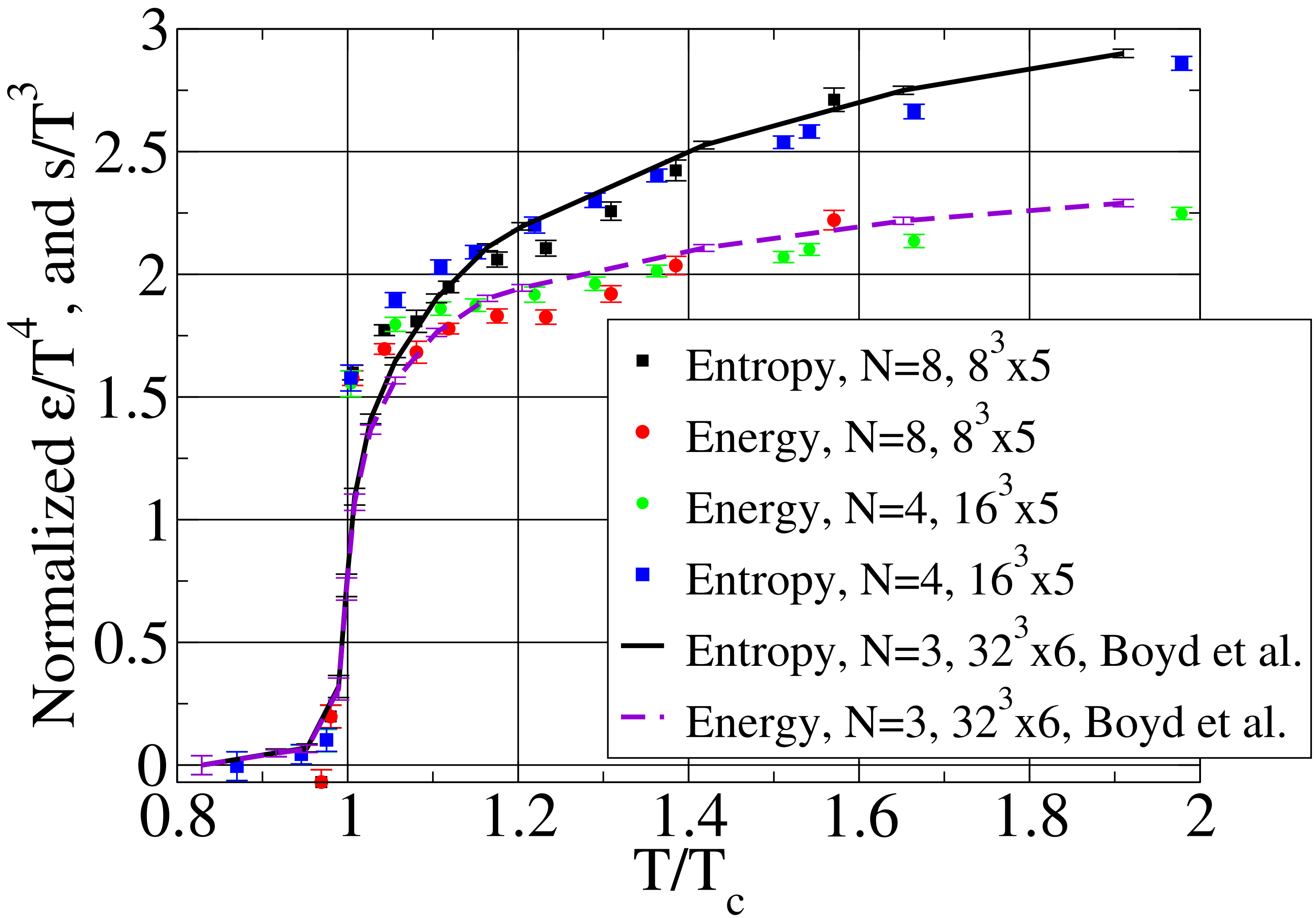
VII.d. Results-Normalized Δ, $L_t = 2, 3, 4, 5$ (higher temperatures.)

Here we see Δ up to $T = 2.5T_c$, again modest $O(1/N)$ corrections.



VII.e. Results-Normalized $\epsilon, s,\ L_t = 5$

$s/T^3, \epsilon/T^4$ have modest $O(1/N)$ corrections.



IX. Summary and Implications (very shortly)

We calculated $\boldsymbol{p, \Delta, \epsilon, s}$ at lattice spacing $a^{-1} = 5T$ with the integral method:

- $\boldsymbol{T \leq 2T_c}$ **for** $\boldsymbol{SU(4)}$.
- $\boldsymbol{T \leq 1.6T_c}$ **for** $\boldsymbol{SU(8)}$.

We calculated $\boldsymbol{\Delta}$ **for** $\boldsymbol{SU(4)}$ **and** $\boldsymbol{SU(8)}$ $\boldsymbol{T_c \leq T \leq 2.5T_c}$ at $a^{-1} = (2-5)T_c$, .

We find that for all quantities the $\boldsymbol{\mathcal{O}(1/N)}$ **corrections are modest/small.**

An exception is with Δ (and therefore also ϵ, s): where the **discontinuity at** $\boldsymbol{T_c}$ **is sharper for** $\boldsymbol{SU(4,8)}$ **than for** $\boldsymbol{SU(3)}$.

Any calculation of $\boldsymbol{p}$ **deficit,**$\boldsymbol{\Delta}$ **must also survive the large-**$\boldsymbol{N}$ **limit which is easier to approach analytically**.

Which means ... .

Diagrammatic methods - **Planar diagrams are most important.**

$\rightarrow$ Up to $\mathcal{O}(g^6)$ a lá Kajantie et al. '02 everything is planar ! Non-planar diag. maybe at $\mathcal{O}(g^7)$ or $\mathcal{O}(g^8)$ Schroder priv. comm.

Models (loops, quasi-particles, bound states)- **Models' parameters must have weak N-dependence.**

Singlet excitations have no role, as there are $\mathcal{O}(1)$ of them.

No role for topology: as at large-N and $T > T_c$ have no instantons Lucini et al. '04, Del Debbio et al. '04

QCD vs. **large-N SUSY models.**

$\rightarrow$ the difference is not due to $\mathcal{O}(1/N)$ corrections.

$\rightarrow$ Understanding what SUSY dynamics imply on nonSUSY gauge theory is sufficient in the large-N limit.

Appendix: Results-Normalized Δ for $SU(3)$, $L_t = 2, 3, 4, 5, 6$ dep.

See systematic L_t dep.