Strongly Interacting Matter within a Quasiparticle Perspective with Critical End Point Effects

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- Comparison with Lattice QCD: Taylor expansion coefficients
- Inclusion of CEP (3D Ising) $\longrightarrow$ Toy Models
- EoS and v_2 for RHIC

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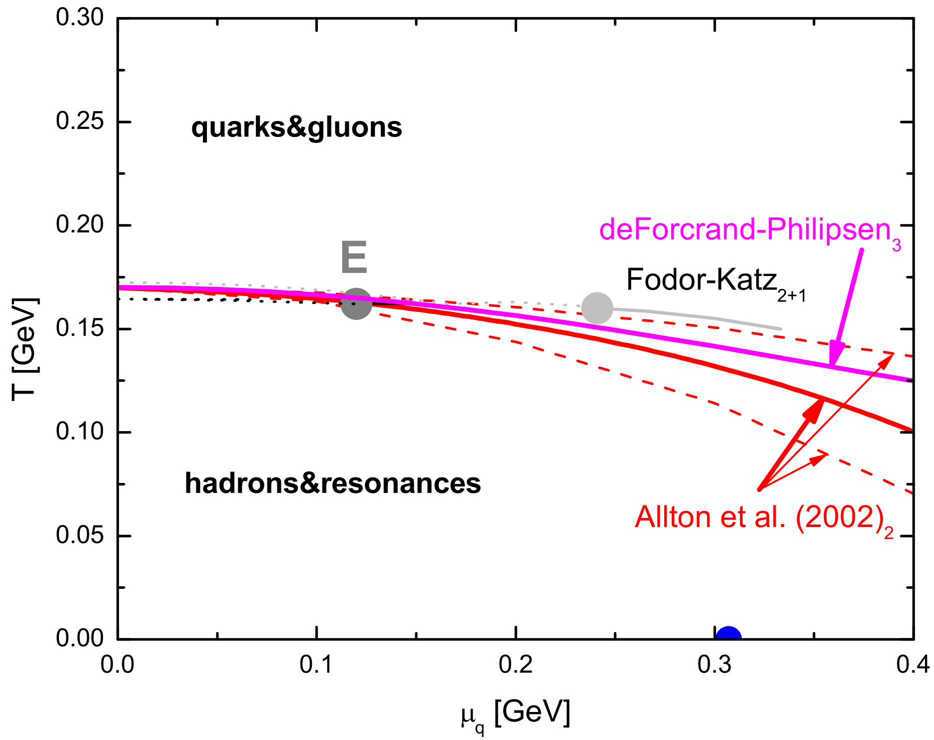
supported by BMBF, GSI, EU

Lattice QCD Results

- Phase Boundary & CEP:



- EoS:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n} \qquad \frac{n_q(T,\mu)}{T^3} = \sum_{n=1}^{\infty} 2n\, c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n-1}$$

$$c_n(T) = \frac{1}{n!} \left. \frac{\partial^n p}{\partial \mu^n} \right|_{\mu=0} \quad \Leftarrow \quad \text{Allton et al. 2003: } N_f = 2 - c_{2,4,6},\ c_0$$

$$m_\pi = 770 \text{ MeV}$$

Quasiparticle Model

thermal equilibrium:

$$p(T,\mu) = \sum_{i=q,g} p_i(T,\mu_i; m_i^2(T,\mu)) - B(m_j^2(T,\mu)) \quad \Rightarrow \quad s = \left.\frac{\partial p}{\partial T}\right|_\mu$$

$$\epsilon = -p + Ts + \mu n$$

$B(T,\mu)$: thermodynamic self-consistency $\Rightarrow$ stationarity condition: $\frac{\delta p}{\delta m_j^2} = 0$

QCD - roots:

- 2-loop $\Phi-$ functional for QCD
- neglect long. gluon mode $\&$ abnormal plasmino branch
- HTL resummed selfenergies $\rightarrow$ gauge inv. $\&$ uv finiteness
- neglect imaginary parts $\&$ Landau damping
- approximate selfenergies at $k \sim T, \mu$

effective coupling at $\mu = 0$:

$$G^2(T) = \begin{cases} G^2_{2-\text{loop}}(\zeta),\ \zeta = \lambda \frac{T-T_s}{T_c}, & T \geq T_c \\ G^2_{2-\text{loop}}(T_c) + b(1-\frac{T}{T_c}), & T < T_c \end{cases}$$

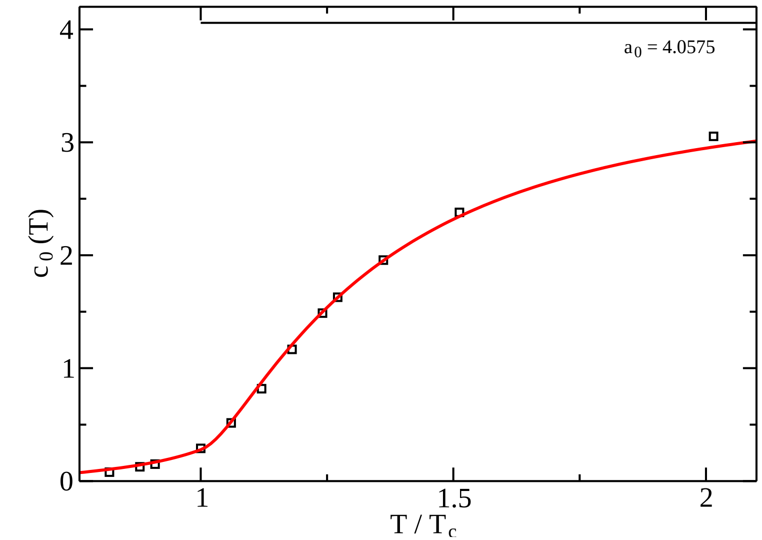
extension to $\mu \neq 0$:

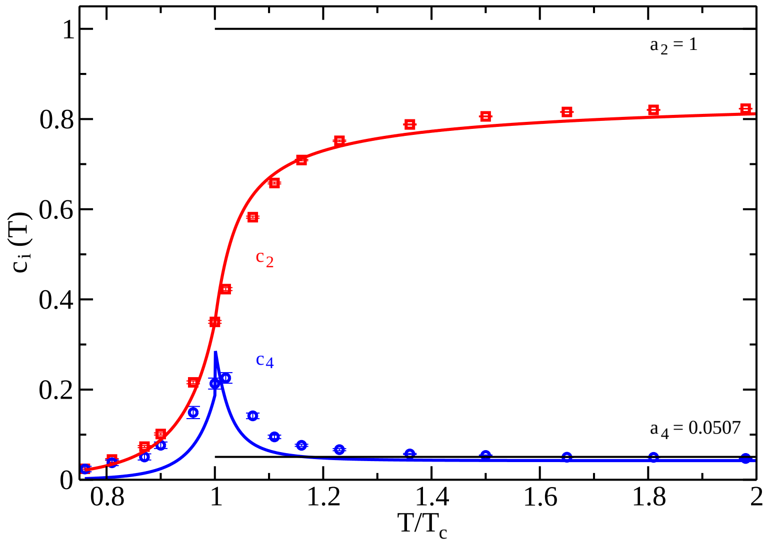
$$a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

$\Rightarrow$ Cauchy-problem: $G^2(T,\mu=0)$

Comparison with Lattice QCD:

Taylor expansion coefficients $c_i(T)$





$$G^2(T) \Longleftrightarrow c_0$$

$$c_2 \propto \int dk \ \ldots G^2 \Big|_{\mu=0}$$

$$c_4 \propto \int dk \left\{ \ldots G^2 + \ldots \frac{\partial^2 G^2}{\partial \mu^2} \right\} \Big|_{\mu=0}$$

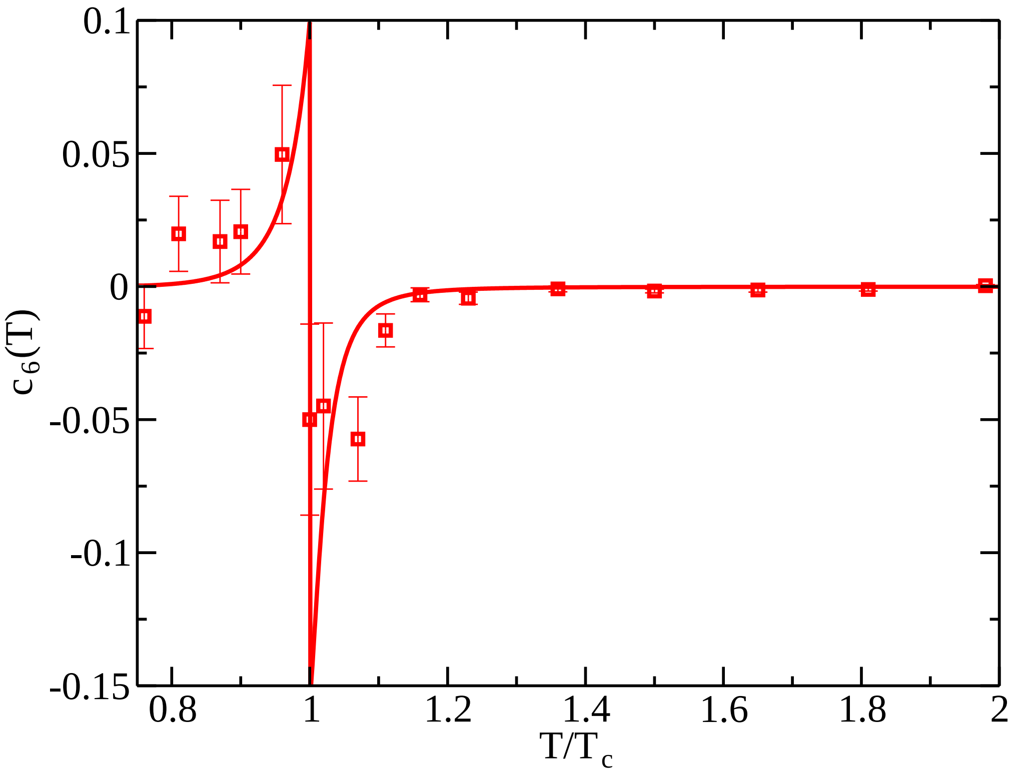
$$G^2(T) \Longleftrightarrow c_2$$

peak in c_4 caused by

$$\frac{\partial^2 G^2}{\partial \mu^2}\Big|_{\mu=0}$$

Taylor Expansion Coefficient $c_6(T)$

$$c_6 \propto \int dk \left\{ \ldots G^2 + \ldots \frac{\partial^2 G^2}{\partial \mu^2} + \ldots \frac{\partial^4 G^2}{\partial \mu^4} \right\}\Bigg|_{\mu=0}$$



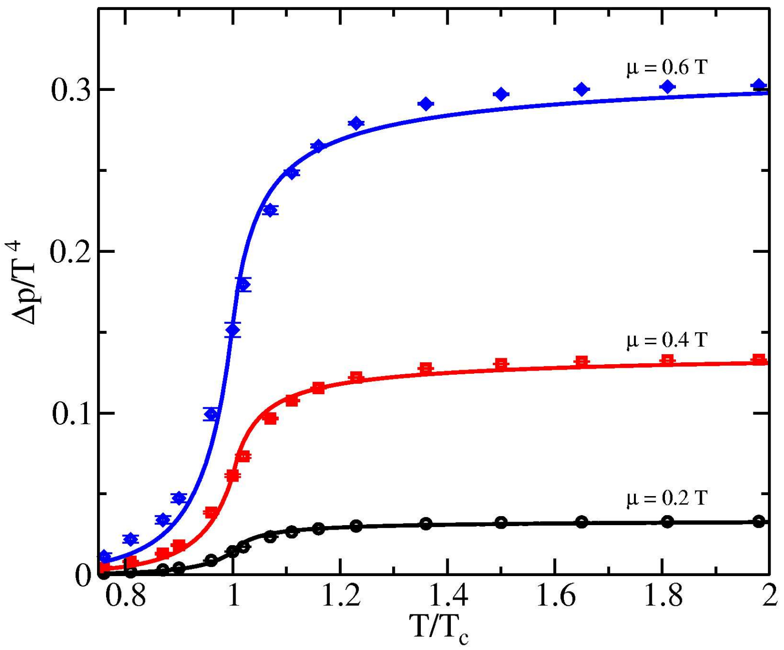
double-peak caused by

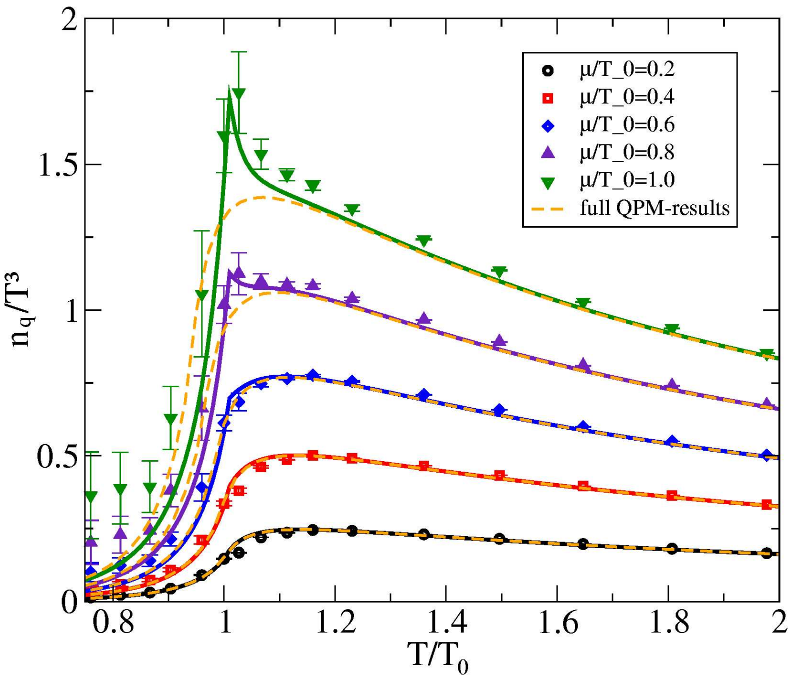
$$\left.\frac{\partial^4 G^2}{\partial \mu^4}\right|_{\mu=0}$$

M. Bluhm et al. PLB 2005

Comparison with Lattice QCD:

Excess pressure and quark number density





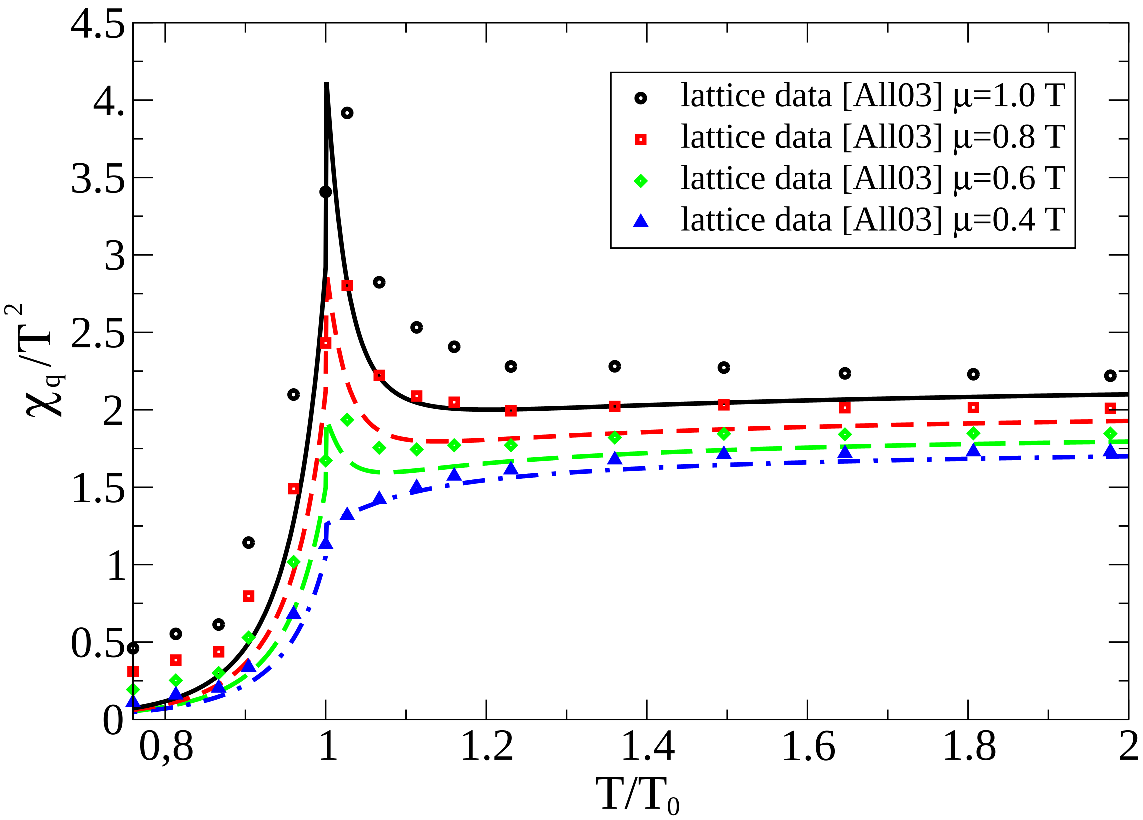
$$T_0 \equiv T_c(\mu_B = 0)$$

$N_f = 2$ – Bielefeld-Swansea

Comparison with Lattice QCD:

Quark number susceptibility

$$\chi_q = \frac{\partial n_q}{\partial \mu}$$



$N_f = 2$ – Bielefeld-Swansea

lQCD:

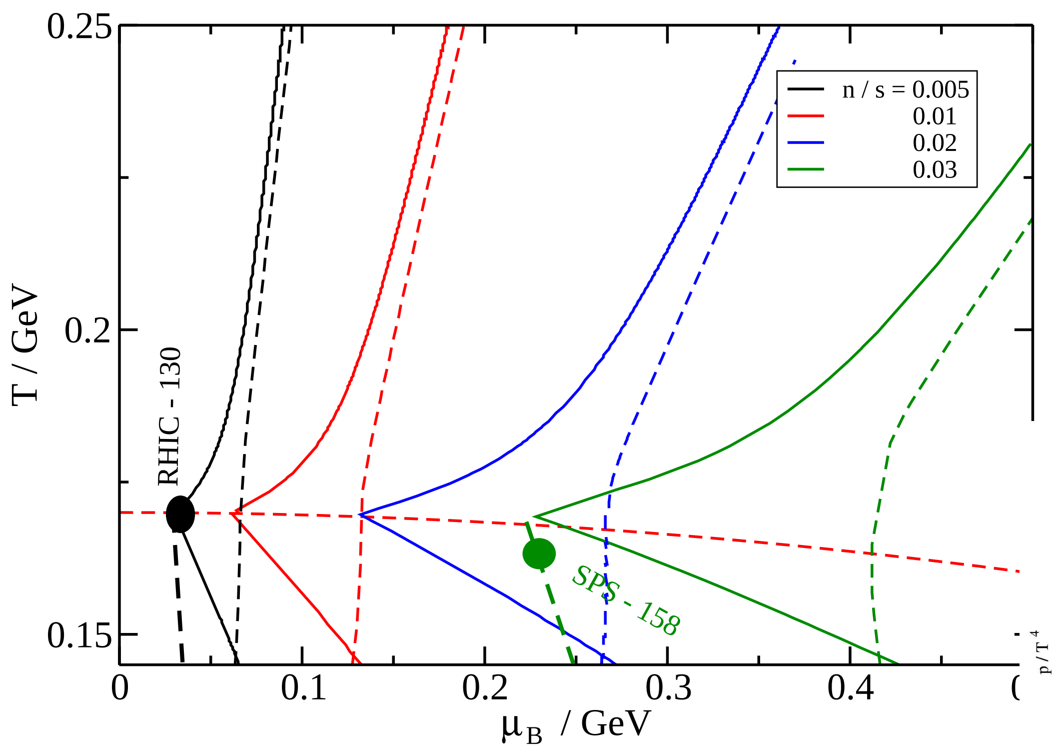
$$\frac{\chi_q}{T^2} = 2c_2 + 12c_4 \left(\frac{\mu}{T}\right)^2$$

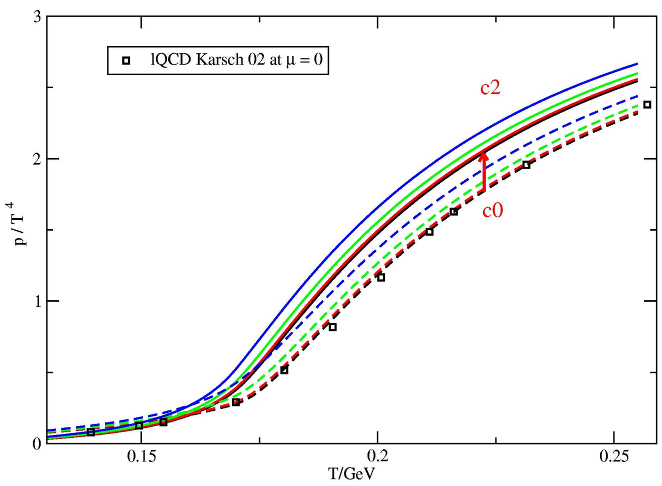
$c_2 \quad \Rightarrow$

Indication for some critical behaviour

Isentropic Expansion

chemical freeze-out: $(\mu_B, T) \quad \rightarrow \quad \frac{s}{n}$





Inclusion of the CEP

Universality hypothesis: QCD Critical point $\Rightarrow$ 3D Ising model

order parameter: magnetisation M(r,h)

$r = \frac{T - T_c}{T_c}$ reduced temperature

h magnetic field

critical point: $(r, h) = (0, 0)$

QCD: $(T, \mu_B) \leftrightarrow (r, h)$

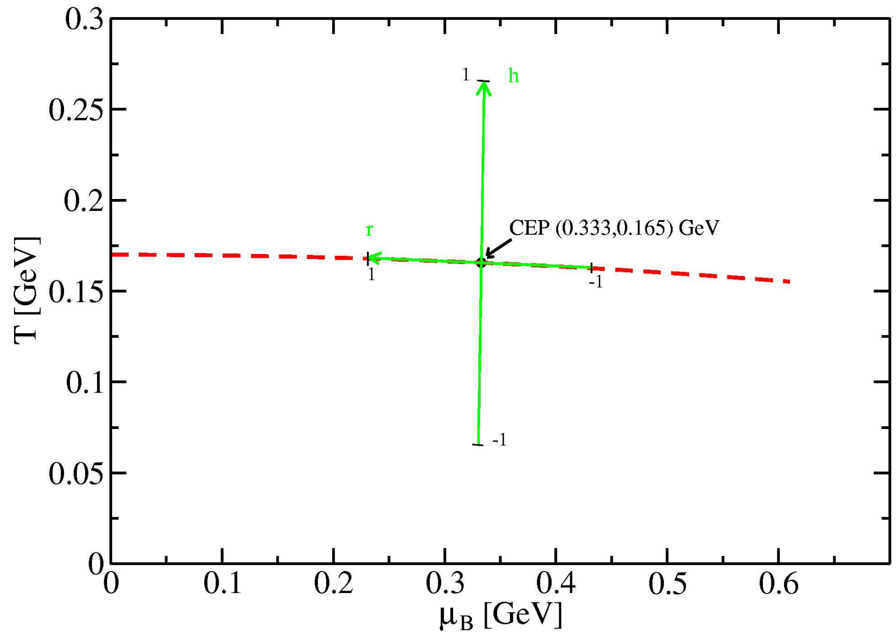
$$r = \frac{(\mu_{B,E} - \mu_B)\cos\phi + (T - T_E)\sin\phi}{\Delta\mu_{B,crit}}$$

$$h = \frac{(\mu_B - \mu_{B,E})\sin\phi + (T - T_E)\cos\phi}{\Delta T_{crit}}$$

scaling:

$\Delta r = 1 \quad \Leftrightarrow \quad \Delta\mu_{B,crit}$

$\Delta h = 1 \quad \Leftrightarrow \quad \Delta T_{crit}$



estimated phase boundary:

$$T_c(\mu_B) = T_c\left(1 + c\left(\frac{\mu_B}{T_c}\right)^2\right)$$

$c = -0.14(6)$ Allton et al. 2002

$c = -0.122$ QPM

Parametric Form of non-analytic Part

3D Ising model - parameters R and θ:
R. Guida and J. Zinn-Justin (1997)

$$(r, h) \leftrightarrow (R, \theta)$$

$$M = M_0 R^\beta \theta$$
$$h = h_0 R^{\beta\delta} \tilde{h}(\theta)$$
$$\tilde{h}(\theta) = \theta + a\theta^3 + b\theta^5$$
$$r = R(1 - \theta^2)$$

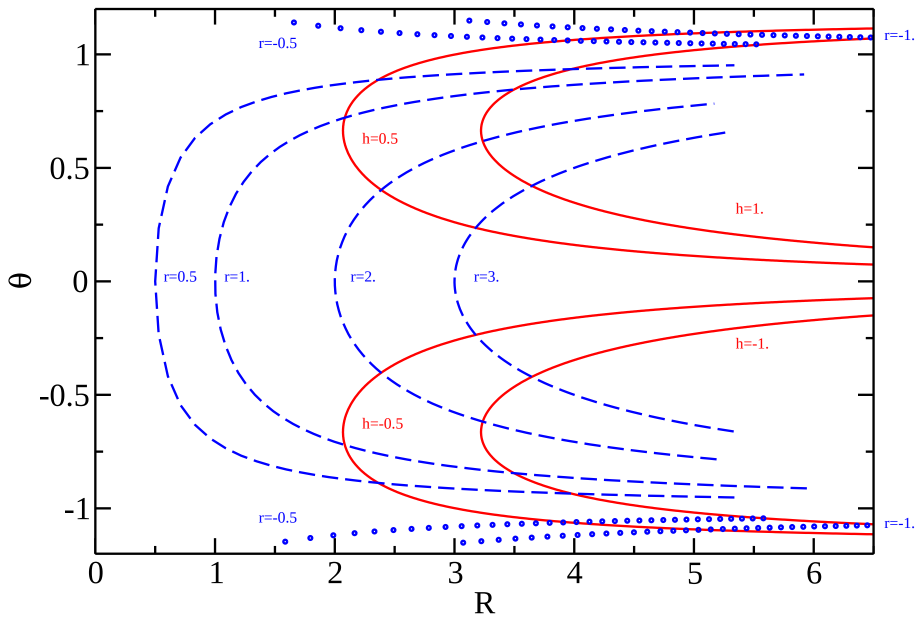
$R \geq 0, -1.154 \leq \theta \leq 1.154$

$a = -0.76201$, $b = 0.00804$

critical exponents: $\beta = 0.326$
$\delta = 4.80$

normalisation: M_0 , h_0

$M(r = -1, h = +0) = 1$
$M(r = 0, h = 1) = 1$



$\Rightarrow$ correct critical behaviour: $M(r = 0, h) \sim \mathrm{sgn}(h)|h|^{1/\delta}$
$M(r, h = +0) \sim |r|^\beta$

Non-analytic Entropy Contribution

C. Nonaka and M. Asakawa (2005)

$G(r,h) = F(M,r) - Mh$

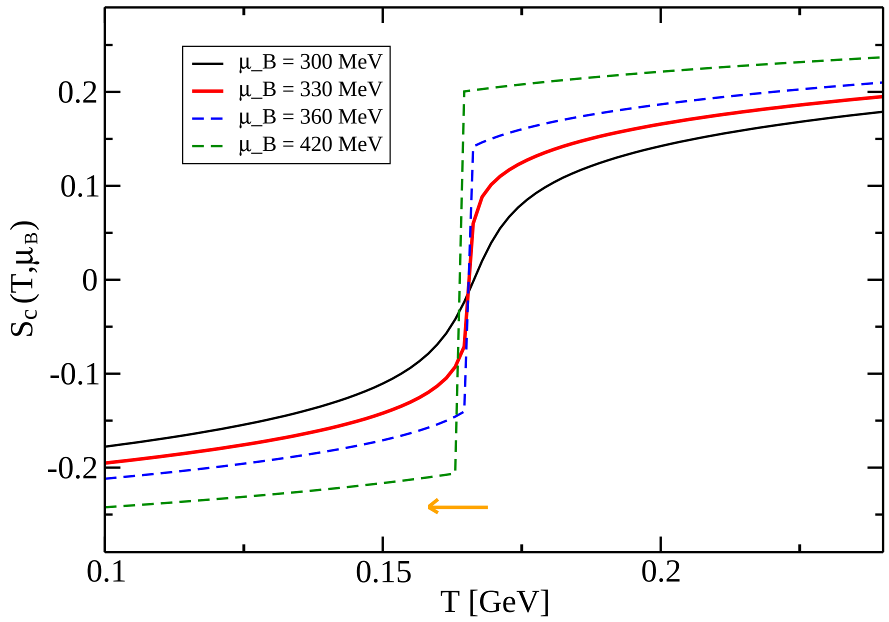
$\Rightarrow F(M,r) = h_0 M_0 r^{2-\alpha} g(\theta)$

$$h = \left(\frac{\partial F}{\partial M}\right)_r \Leftrightarrow \left(\frac{\partial G}{\partial M}\right)_{r,h} = 0$$

$2 - \alpha = \beta\delta + \beta$

$\gamma = \beta(\delta - 1)$

(Essam-Fisher, Widom)



near critical point:

$$s_c = -\left(\frac{\partial G}{\partial T}\right)_{\mu_B}$$

$$= -\left(\frac{\partial G}{\partial h}\right)_r \left(\frac{\partial h}{\partial T}\right)_{\mu_B} - \left(\frac{\partial G}{\partial r}\right)_h \left(\frac{\partial r}{\partial T}\right)_{\mu_B}$$

connection with analytic contribution: $[s_c] = (energy)^{-1}$

$$S_c(T,\mu_B) = D\left(\Delta T_{crit}^2 + \Delta\mu_{B,crit}^2\right)^{1/2} s_c(T,\mu_B)$$

Toy Model I: $a_{0,2,4} = const$

$$s(T,\mu_B) = s_{reg}(T,\mu_B)\left(1+\tilde{s}_{sing}(T,\mu_B)\right)$$

$$s_{reg} = 4a_0T^3 + 2a_2\left(\frac{\mu_B}{3}\right)^2 T$$

$$\Rightarrow c_H = T\left.\frac{\partial s}{\partial T}\right|_{\mu_B} \longrightarrow |T-T_c|^{-\alpha}$$

with

$$a_0 = \frac{32+21N_f}{180}\pi^2$$

$$a_2 = \frac{N_f}{2}$$

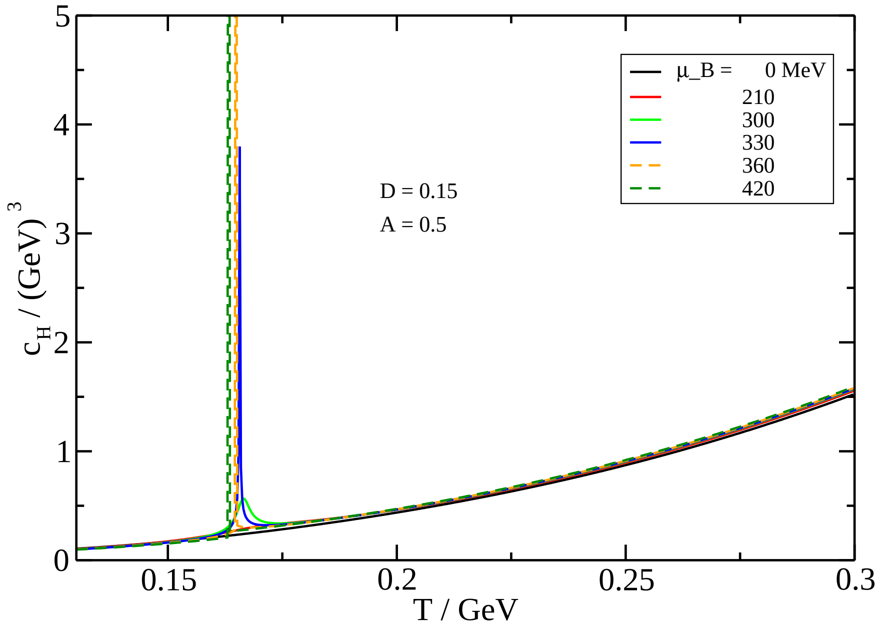
$$\tilde{s}_{sing} = A\tanh S_c(T,\mu_B)$$

$\Delta T_{crit} = 100$ MeV

$\Delta\mu_{B,crit} = 200$ MeV

$$n_B = \int_0^T \frac{\partial s}{\partial\mu_B}dT' + \frac{4}{3}a_4\left(\frac{\mu_B}{3}\right)^3$$

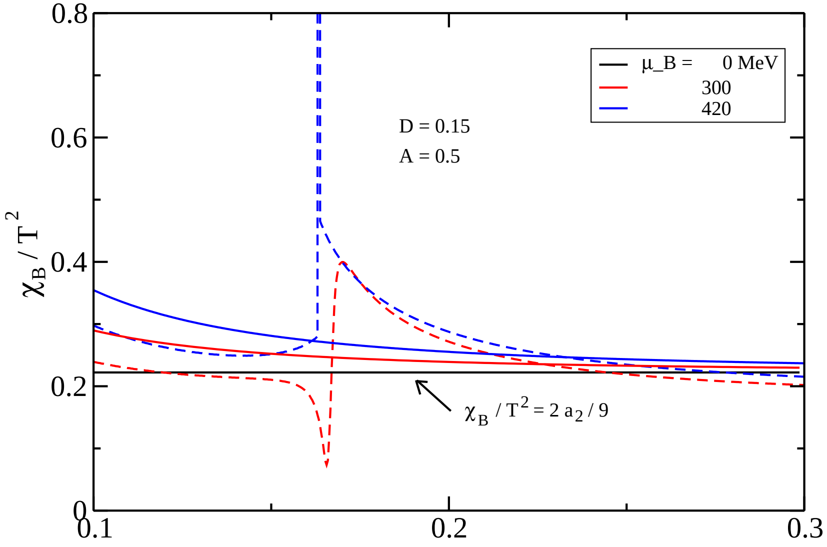
$$a_4 = \frac{N_f}{4\pi^2}$$

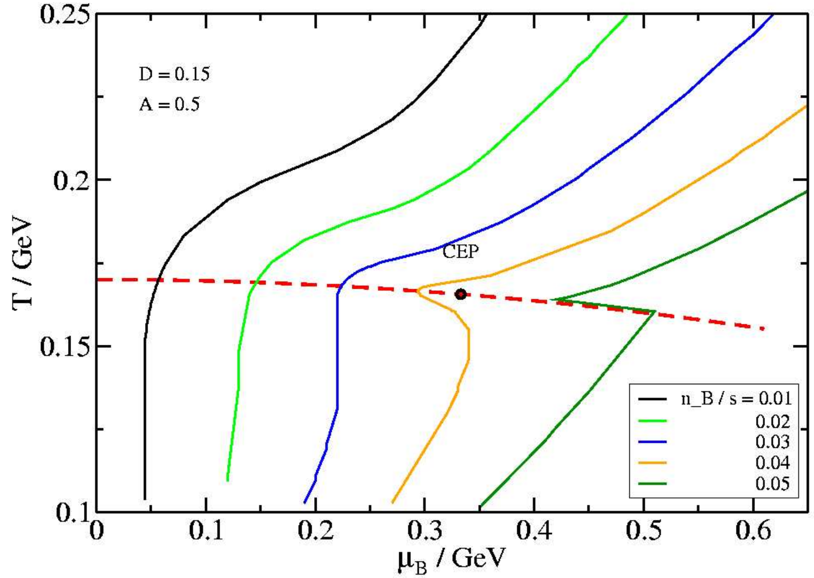


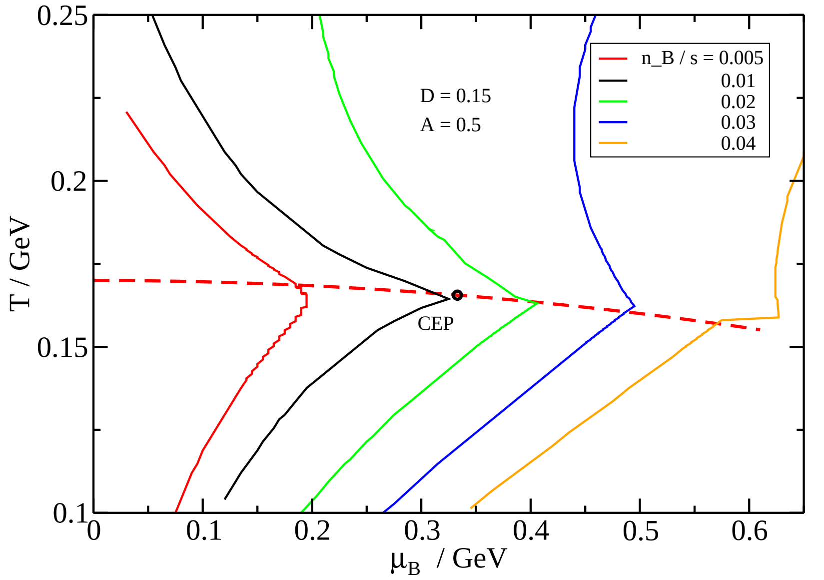
step for $\mu_B > \mu_{B,E}$ and $T > T_c(\mu_B)$:

$$\left|\frac{\partial T_c(\mu_B)}{\partial\mu_B}\right|\left[s(T_c(\mu_B)+0,\mu_B) - s(T_c(\mu_B)-0,\mu_B)\right]$$

CEP: Attractor - Repulsor







$A < 0$

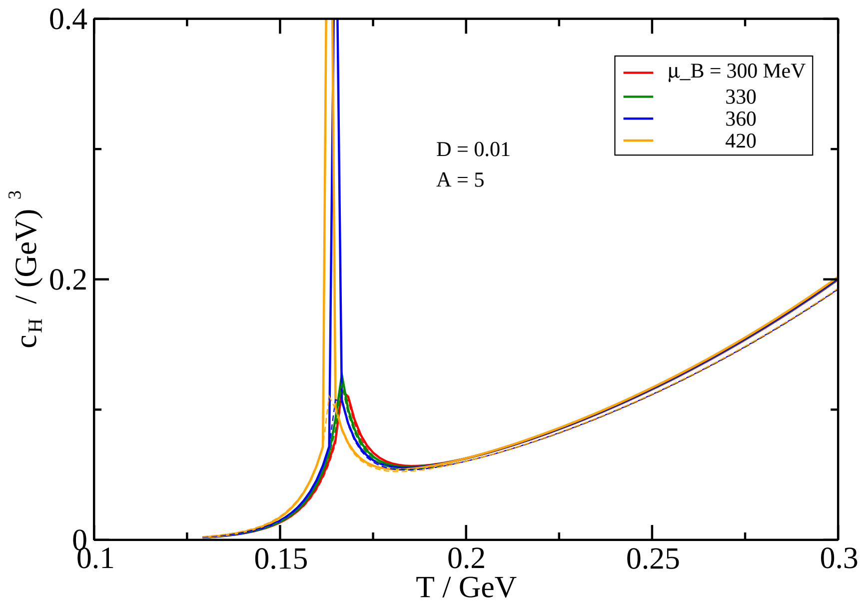
$\longrightarrow$ unphysical

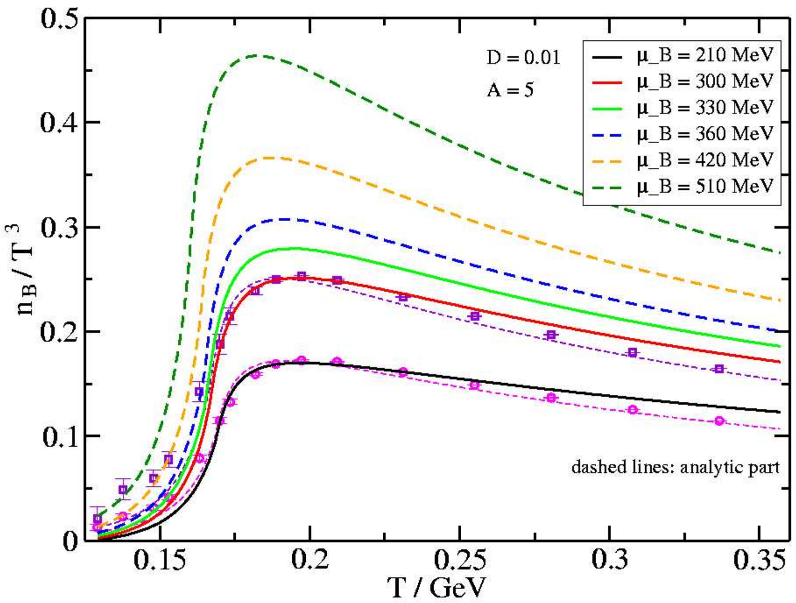
$A > 0$

Toy Model II: QPM with CEP

$$s(T,\mu_B) = s_{QPM}(T,\mu_B) + F(T,\mu_B) A \tanh S_c(T,\mu_B)$$

$$F(T,\mu_B) \sim T^3, \mu_B T^2, \mu_B^2 T, \mu_B^3$$

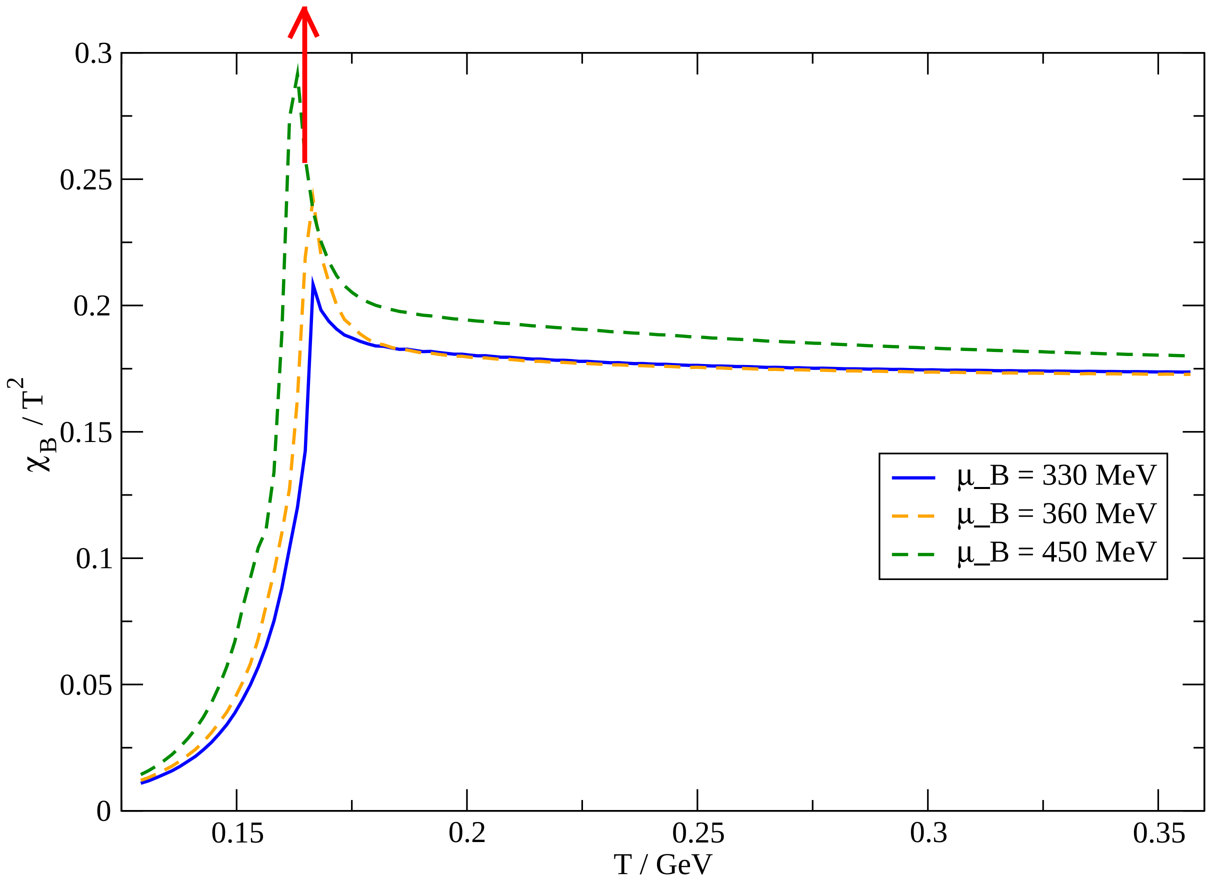




$\Delta T_{crit} = 50$ MeV
$\Delta\mu_{B,crit} = 50$ MeV

$\rightarrow$ small modification

Susceptibility

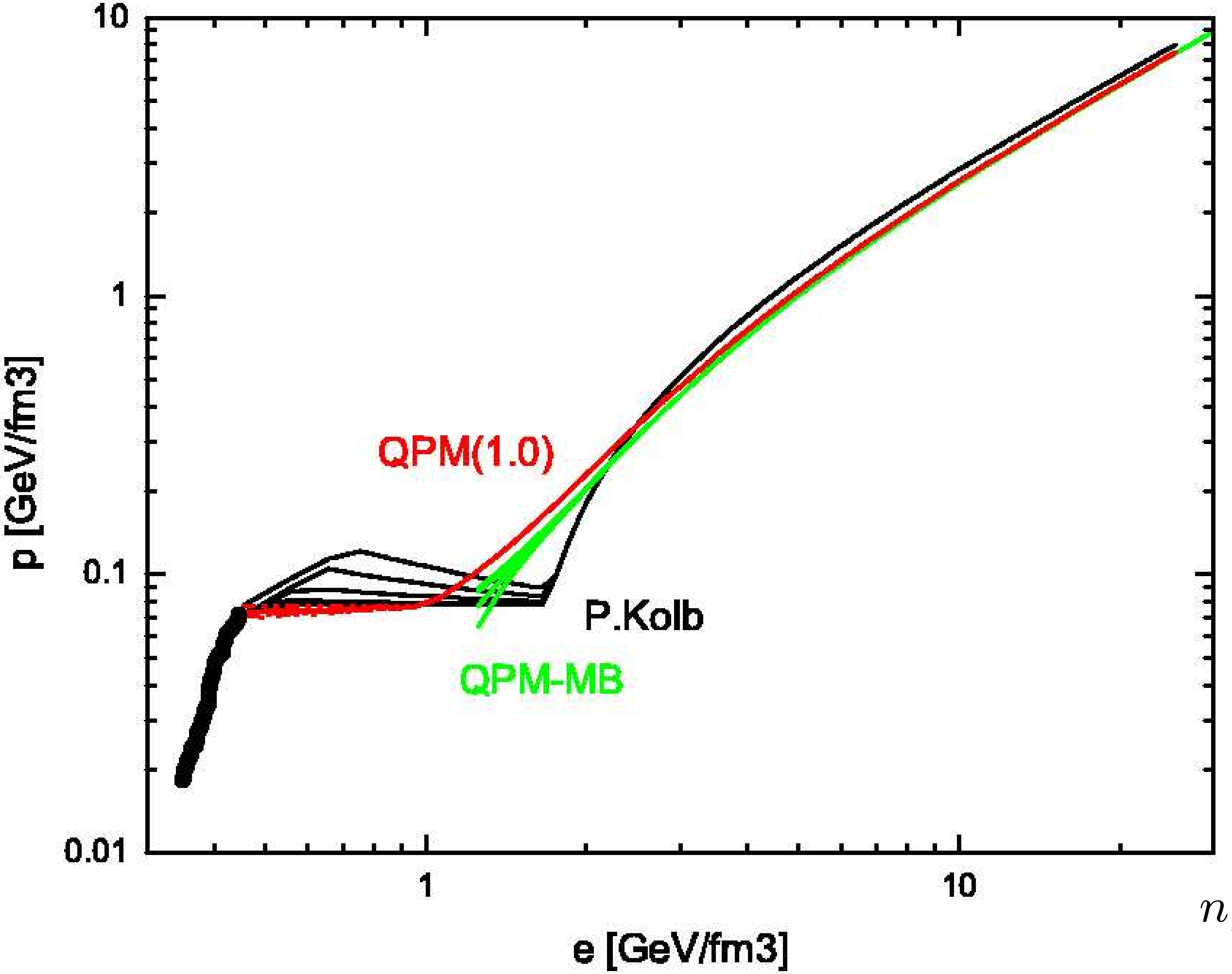


χ_B: Singular behaviour

$\Rightarrow$ Conjecture: hydro-evolution of $v_2, \ p_T$ at RHIC independent of CEP !

Equation of State

Progress of lQCD:
High-density part fixed

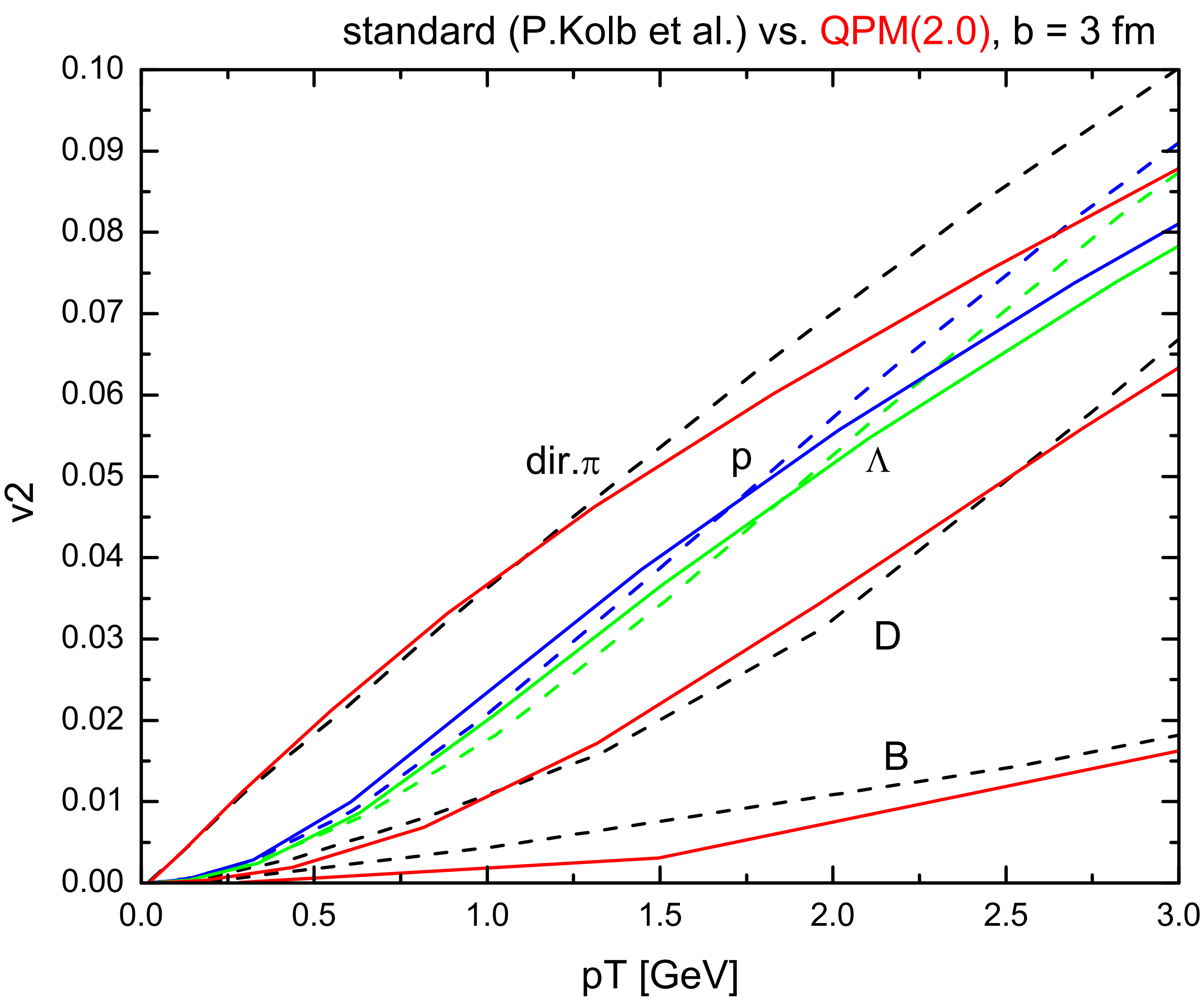


with
U. Heinz/Ohio

tiny baryon
density effects
$n_B = 0...0.4 fm^{-3}$

Progress of lQCD:
Low-density part fixed

Azimuthal Anisotropy



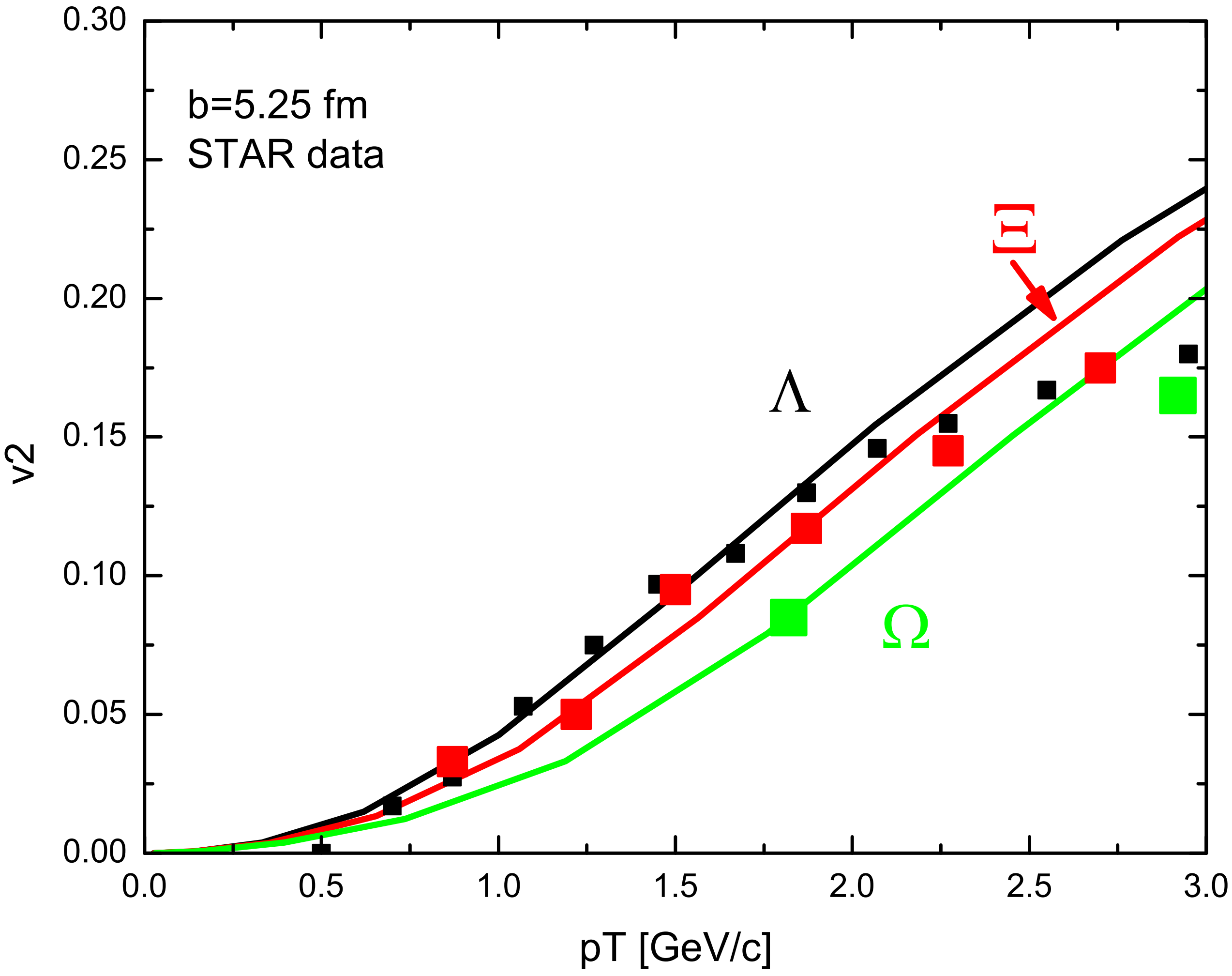
⇒ weak dependence of v_2 on EoS

Conclusion & Outlook

- QPM describes lattice data: $N_f = 2$
- expansion coefficients $c_i(T)$: $c_2 \quad \longrightarrow \quad c_4,\, c_6$

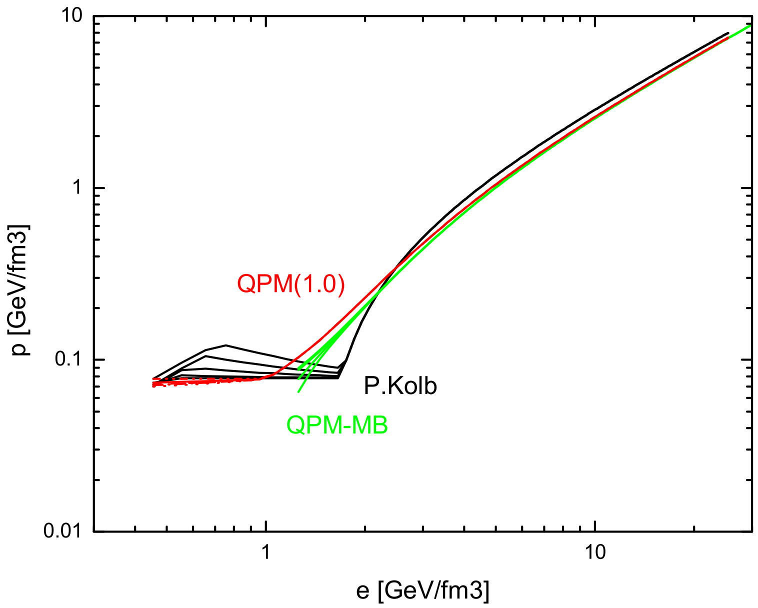
  pronounced behaviour at $T_c \quad \Longleftarrow \quad \left.\frac{\partial^n G^2}{\partial \mu^n}\right|_{\mu=0}$
- deviation: $c_0 \quad \Leftrightarrow \quad c_2$
- first hints for consistency of chemical freeze-out and isentropes
- Toy Models with CEP:
  - many free parameters
  - singular behaviour in observables
- v_2: RHIC $\Rightarrow$ EoS at T_c unimportant
- CERN SPS, CBM-FAIR: very different

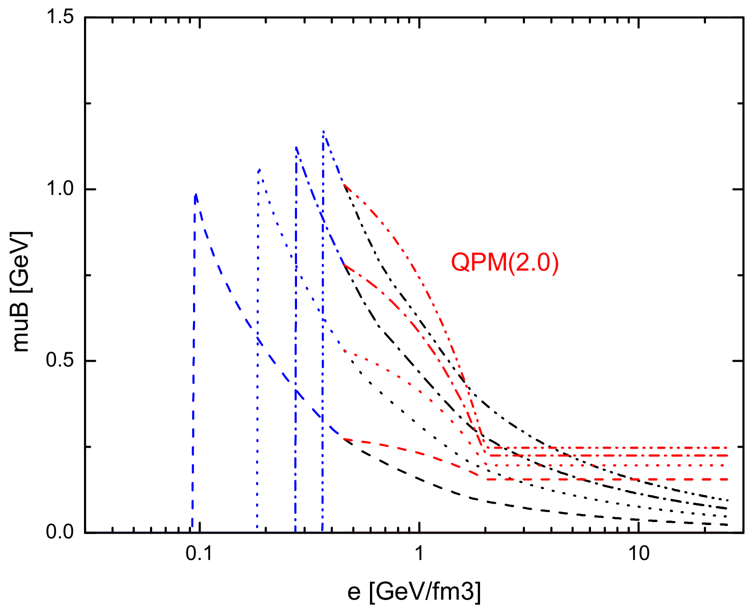
EoS - Aside I

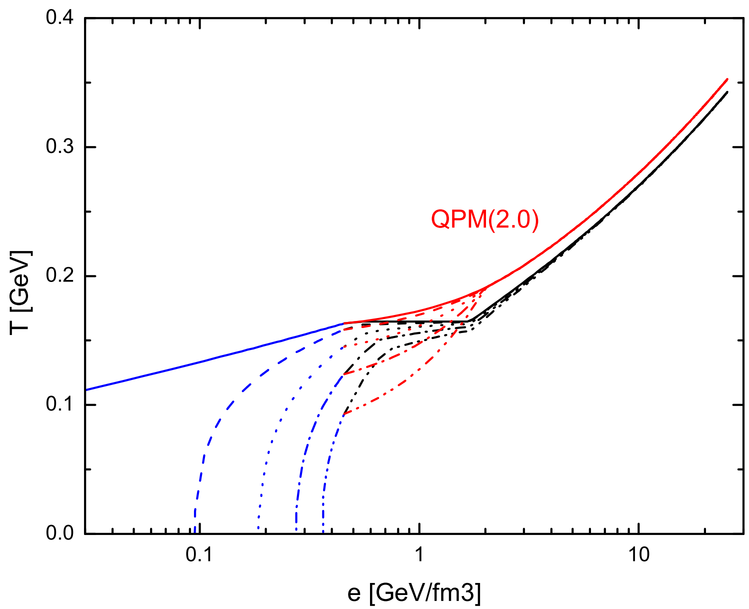


b=5.25 fm
STAR data
Λ
Ξ
Ω
v2
pT [GeV/c]
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.0
0.5
1.0
1.5
2.0
2.5
3.0

EoS - Aside II


10
1
0.1
0.01
p [GeV/fm3]
1
10
e [GeV/fm3]
QPM(1.0)
P.Kolb
QPM-MB


1.5
1.0
0.5
0.0
muB [GeV]
0.1
1
10
e [GeV/fm3]
QPM(2.0)


0.4
0.3
0.2
0.1
0.0
T [GeV]
0.1
1
10
e [GeV/fm3]
QPM(2.0)