$N_C = 2$ Lattice QCD with dynamical fermions at non-zero density versus Matrix Models

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references: hep-lat/0409045; hep-th/0507156

> **Aim:** **Matrix Models describe dynamical fermions + chemical potential**

- **here:** **two-colour QCD** $N_f = 2$ **staggered flavours** $\mu \neq 0$

  $\Rightarrow$ **no sign problem!**

- **known data vs. MM:**

  $\mu = 0$**:** M.E. Berbenni-Bitsch, S. Meyer, T. Wettig, `hep-lat/9804030`
  G. A., E. Kanzieper, `hep-th/0001188`

  $\mu \neq 0$**: Matrix Model vs. quenched QCD**

  G. A., T. Wettig `hep-th/0308003`

- $\exists$ **Matrix Model predictions for unquenched QCD with sign problem:**

  J.C. Osborn `hep-th/0403131`
  G. A., J.C. Osborn, K. Splittorff, J.J.M. Verbaarschot, `hep-th/0411030`

What is the Matrix Model Approach?

global symmetries:

- $\{\not{D}, \gamma_5\} = 0 \qquad \Rightarrow$ block structure

- $[C(\sigma_2)K, \not{D}] = 0$ for **adjoint / SU(2)** $\quad \Rightarrow$ symmetry of matrix

idea: replace $\not{D} \longrightarrow \begin{pmatrix} 0 & \Phi \\ -\Phi^\dagger & 0 \end{pmatrix}$ $\quad \Phi$ random matrix $\in \mathbb{C} / \mathbb{H} / \mathbb{R}$

for **QCD/ adjoint / SU(2)**

and $\langle \cdots \rangle_{GAUGE} \longrightarrow \langle \cdots \rangle_{MM}$ $\qquad$ [Shuryak, Verbaarschot '93]

[Verbaarschot '94]

- **fixed topology:** $\Phi = \boxed{}$ $\quad$ ("staggered": $\nu = 0$)

+ chemical potential $\qquad \boxed{\begin{array}{c} \not{D} + \gamma_0\mu \\ \textbf{non-Hermitian} \end{array}}$

[Stephanov '96; Osborn '04]

Matrix Model with chemical potential $\mu \neq 0$

$$\mathcal{Z}_{MM} \equiv \int d\Phi d\Psi \; \Pi_{f=1}^{N_f} \det \begin{pmatrix} m_f & i\Phi + \mu\Psi \\ & \\ i\Phi^\dagger + \mu\Psi^\dagger & m_f \end{pmatrix} \mathrm{e}^{-N\langle \bar{q}q \rangle^2 \mathrm{Tr}(\Phi\Phi^\dagger + \Psi\Psi^\dagger)}$$

- compute correlation functions of eigenvalues $z_j^2 \in \mathbb{C}$

  of $(i\Phi + \mu\Psi)(i\Phi^\dagger + \mu\Psi^\dagger)$ $\quad N \times (N+\nu)$ matrices $\in \mathbb{H}$ [G.A. '05]

- $2N \sim$ **Volume**

- <u>here:</u>

  c.c. eigenvalue pairs $\det[\slashed{D}(m) + \mu\gamma_0] = \prod_j^N |z_j^2 + m^2|^2$

  adjoint class with chemical potential

- **staggered fermions: adjoint $\longleftrightarrow$ SU(2) fund. class**

- model for **small** μ and **T (=0)**: **away from transition**

MM = ϵ–Regime of chiral Perturbation Theory?

$$\mathcal{Z}_\varepsilon = \int dU_0 \det[U_0]^\nu \exp\left[\mu^2 F^2 V \operatorname{Tr}(U_0 B^T U_0^\dagger B) + \langle \bar{q}q \rangle V \operatorname{Tr}(M_q(U_0 + U_0^\dagger))\right]$$

[Kogut et al. '00]

$U_0 \in U(N_f)$ zero mode of Goldstone field; $\quad U_0 = U_0^T$ for adj.

$B = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$ baryon charge matrix

validity:
$$1/\Lambda \ll V^{1/4} \ll 1/m_\pi$$
$$\mu^2 V \ll V^{-1/2}$$

to ensure dominance of U_0 ! ($\times$ free fields)

• **have to prove MM $\equiv$ ϵ–chPT for $\mu \neq 0$! QCD**:

[Splittorff,Verbaarschot '03; G.A., Fyodorov, Vernizzi '04]

scaling:
$$\mu^2 F^2 V \quad \& \quad \langle \bar{q}q \rangle V M_q$$

What can be computed from Matrix Models

- **MM** describe **universal properties** of spectra:
  repulsion between energy levels

- spectral correlations of $\not{D}$–eigenvalues: $\rho(z \approx 0) \sim \langle \bar{q}q \rangle$ [Banks,Casher'80]

- **exact expression** $\rho(zV)$ as a function $z \ \in \mathbb{C}$ of

  – **symmetry class QCD/ adj / SU(2)**

  – **topology**

  – **number of flavours** N_f **& masses** M_q

  – **chemical potential** μ

- at $\mu = 0$ parameter free prediction: eliminate $\langle \bar{q}q \rangle \leftrightarrow$ level spacing
  at $\mu \neq 0$ **1 fit parameter** (F not contained in MM)
  $\Rightarrow$ **2 LEC in chPT at** $\mu \neq 0$

Spectral density: quenched, masses, topology
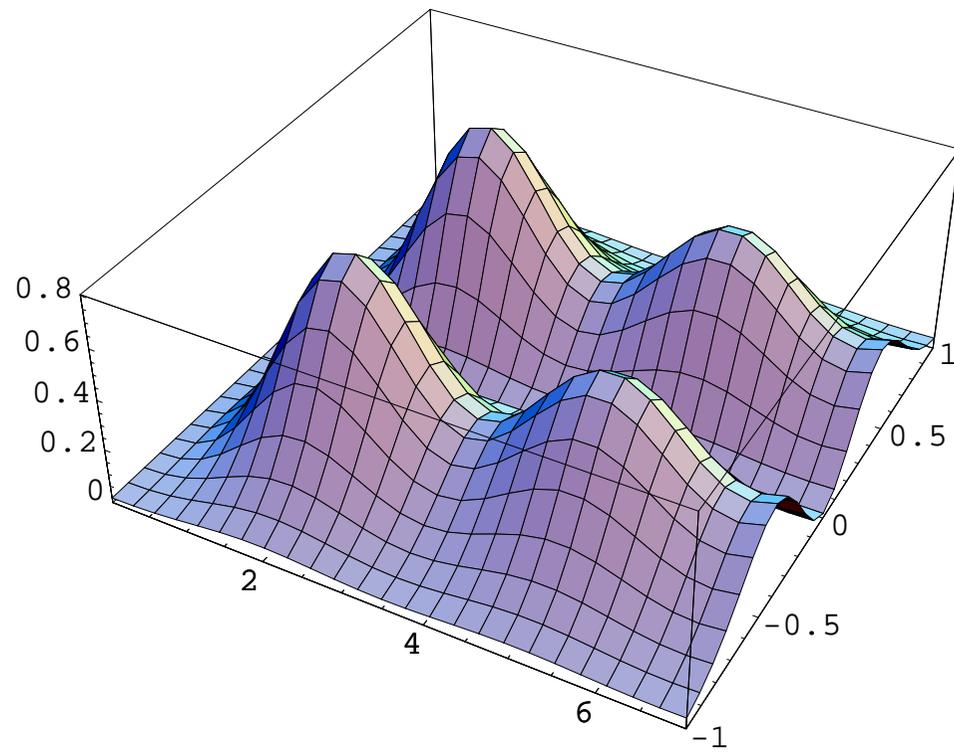
- **quenched density** $\nu = 0$
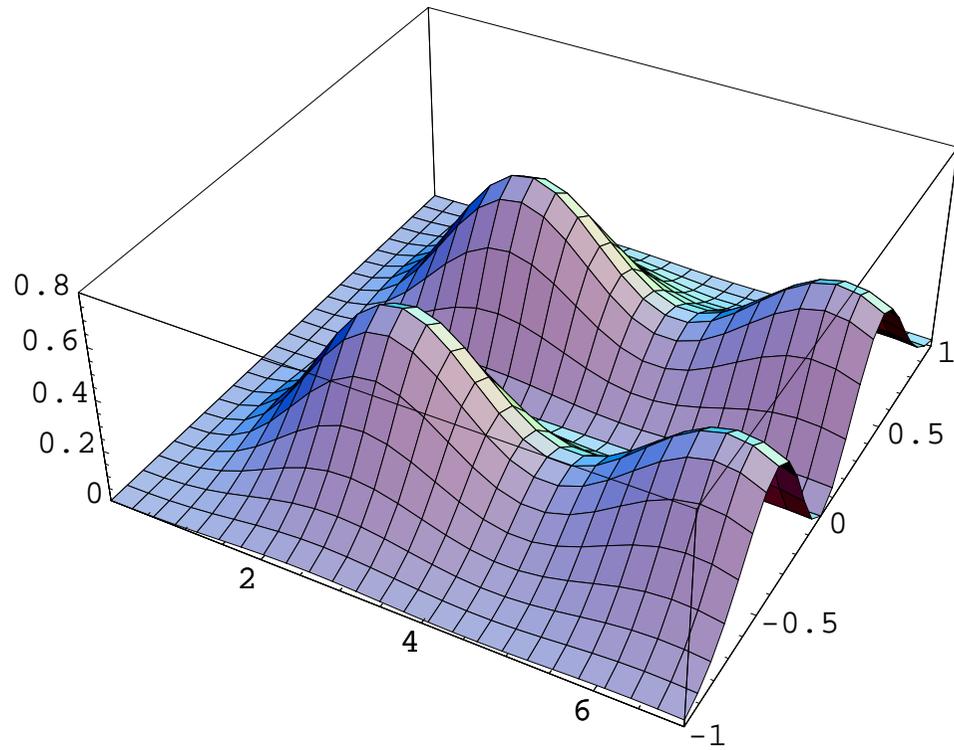
- **massive density** $mV\langle \bar{q}q \rangle \sim 9$
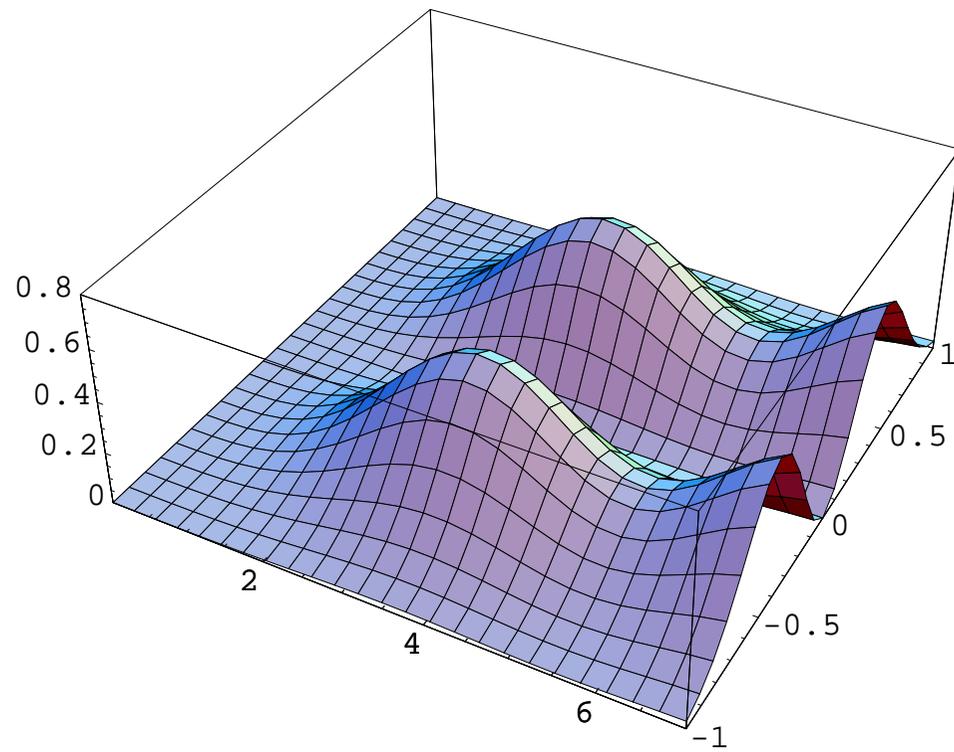
- **massive density** $mV\langle\bar{q}q\rangle \sim 4$
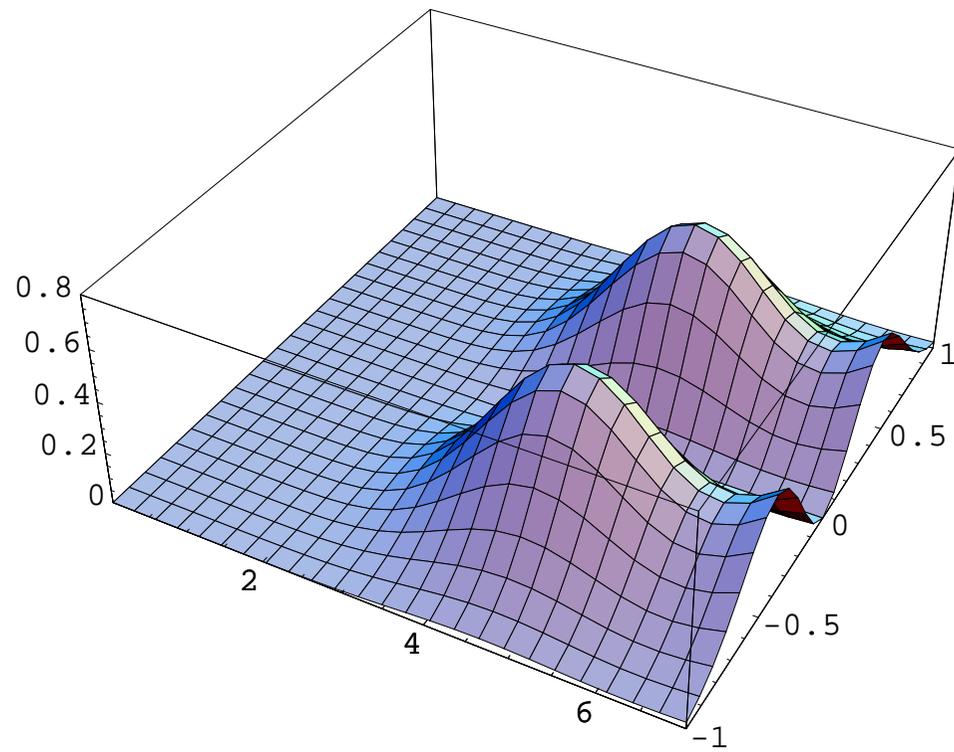
- **quenched density** $\nu = 2$
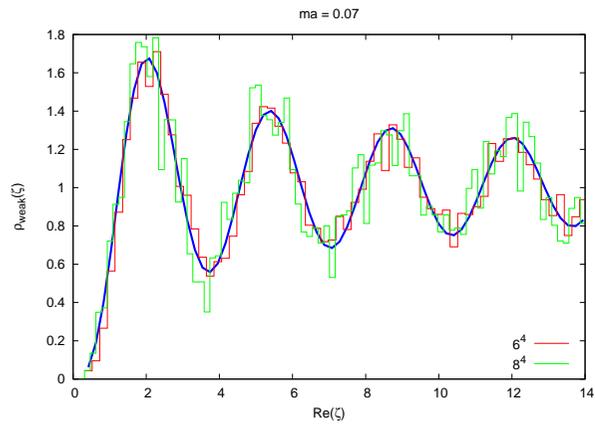
Compared to quenched QCD
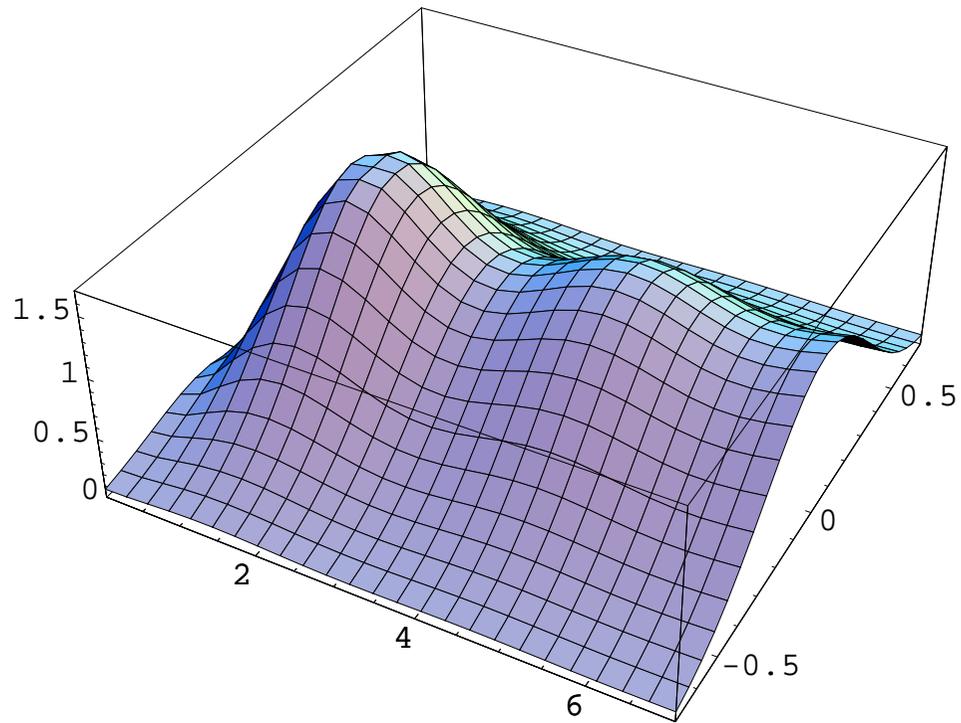
- very different: **no repulsion from axis!**
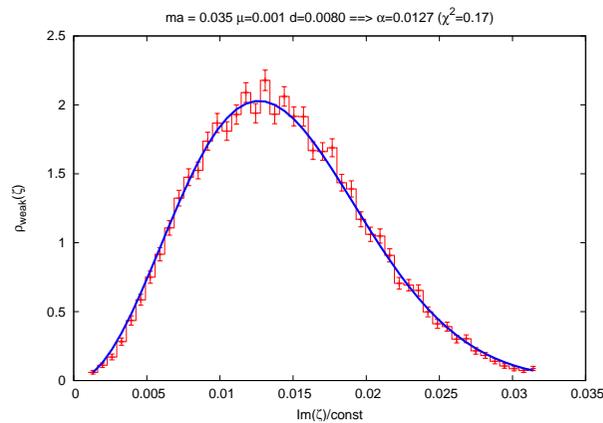
Comparison to SU(2) Lattice: cut curves
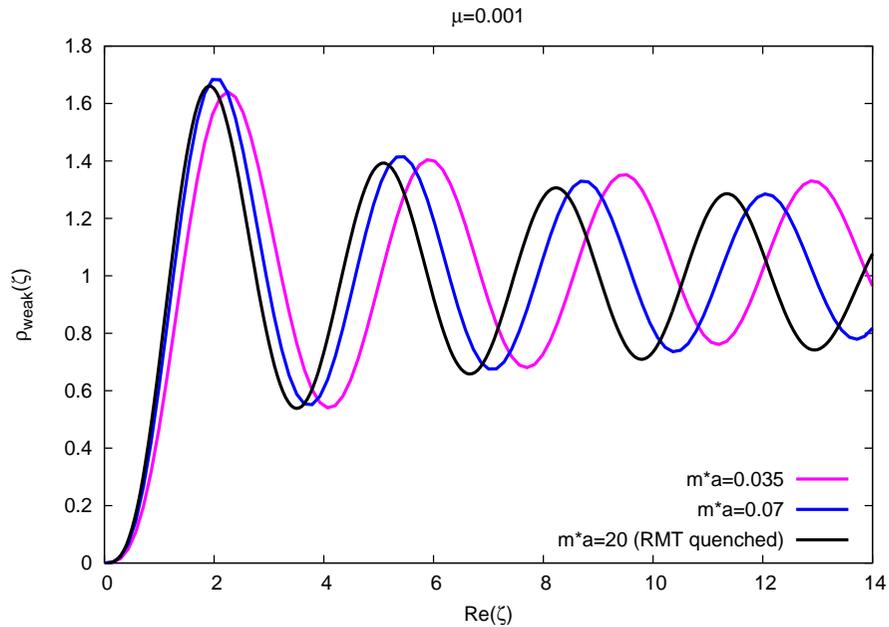
- **effect of** $N_f = 2$ at our parameters:

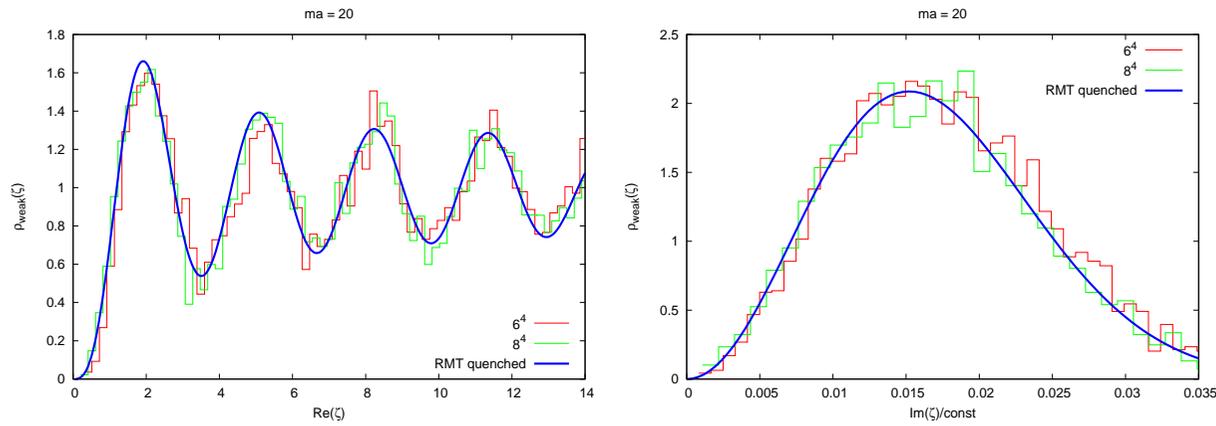 (small) **shift of maxima** in $\Re e$-direction

... versus unquenched data
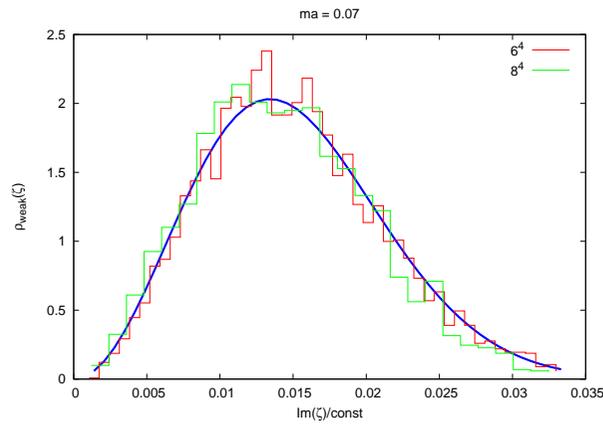
- $ma = 0.07$ **scaling** $\mu^2 V = $ **const.:** $V = 6^4$ vs. 8^4

1 parameter fit (right)
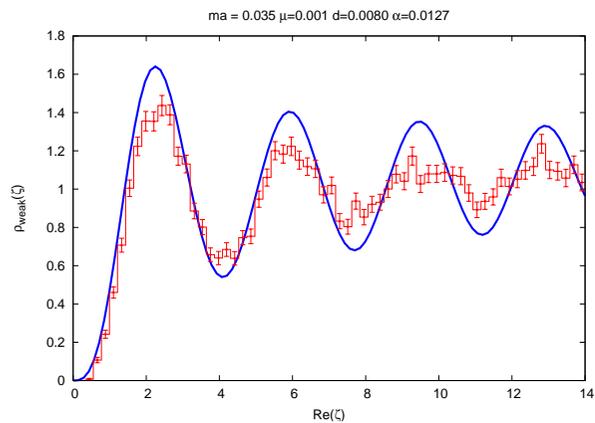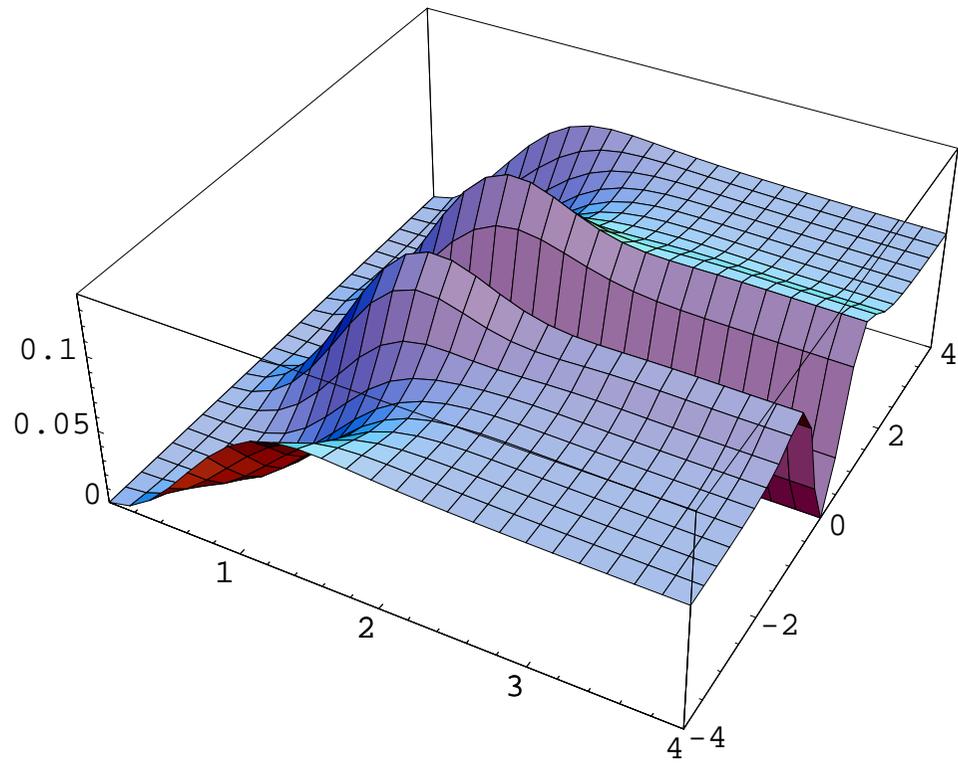

- $ma = 0.0035$ $V = 6^4$

Quenching via large mass

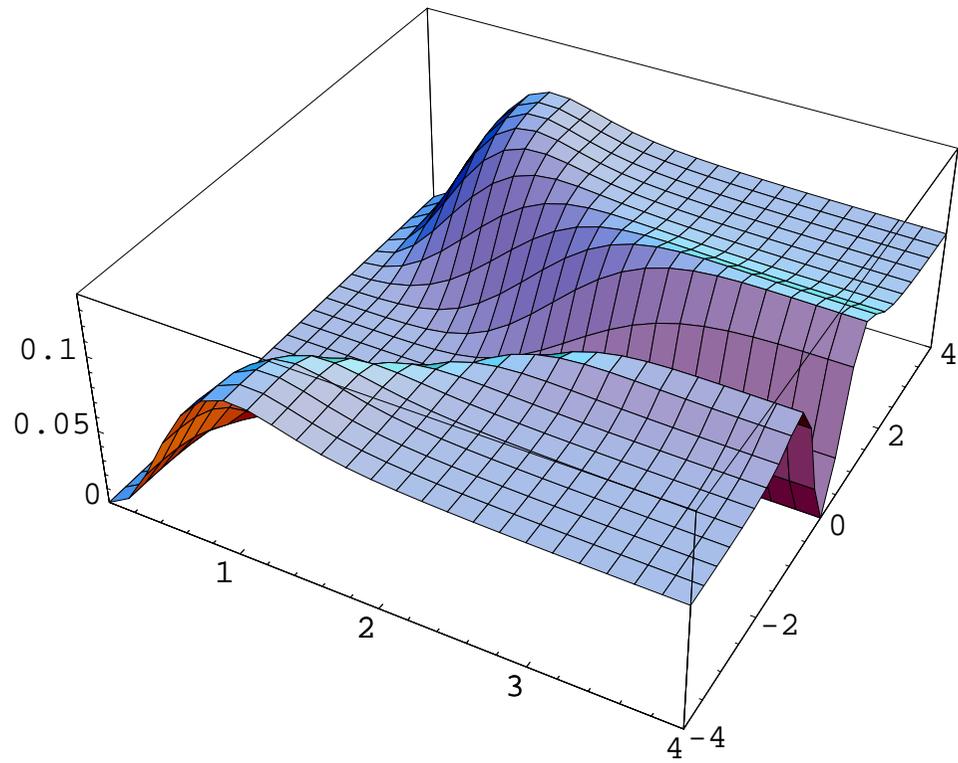- $ma = 20$: **quenched curve, + scaling** at $V = 6^4$ vs. 8^4
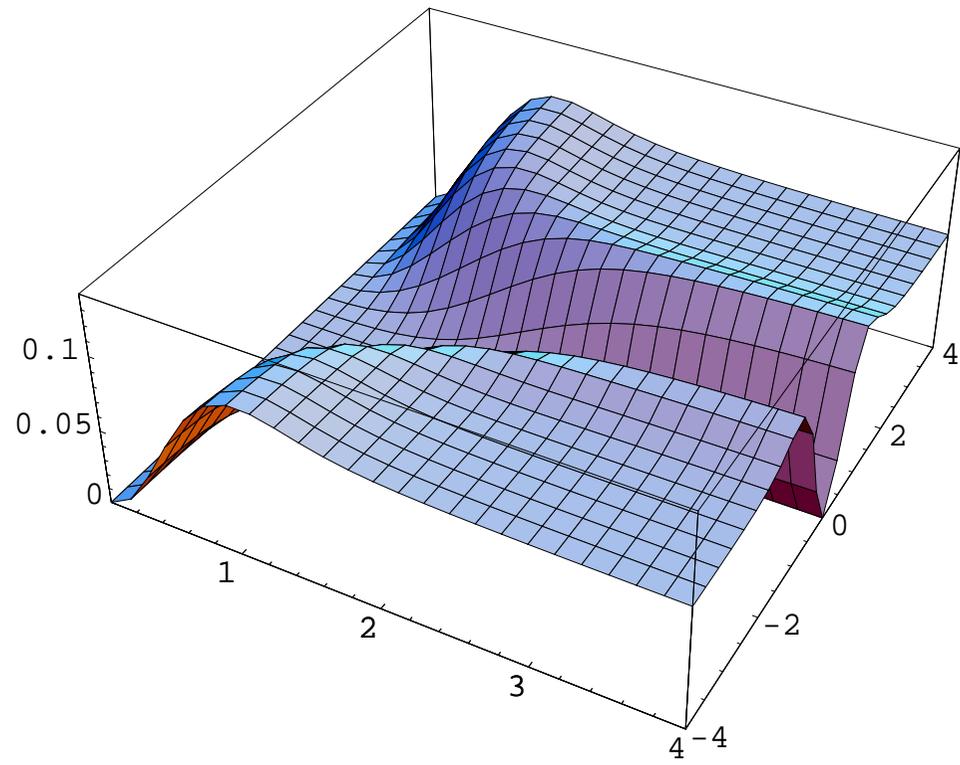
Limit of large $\mu^2 F^2 V$: strong non-Hermiticity
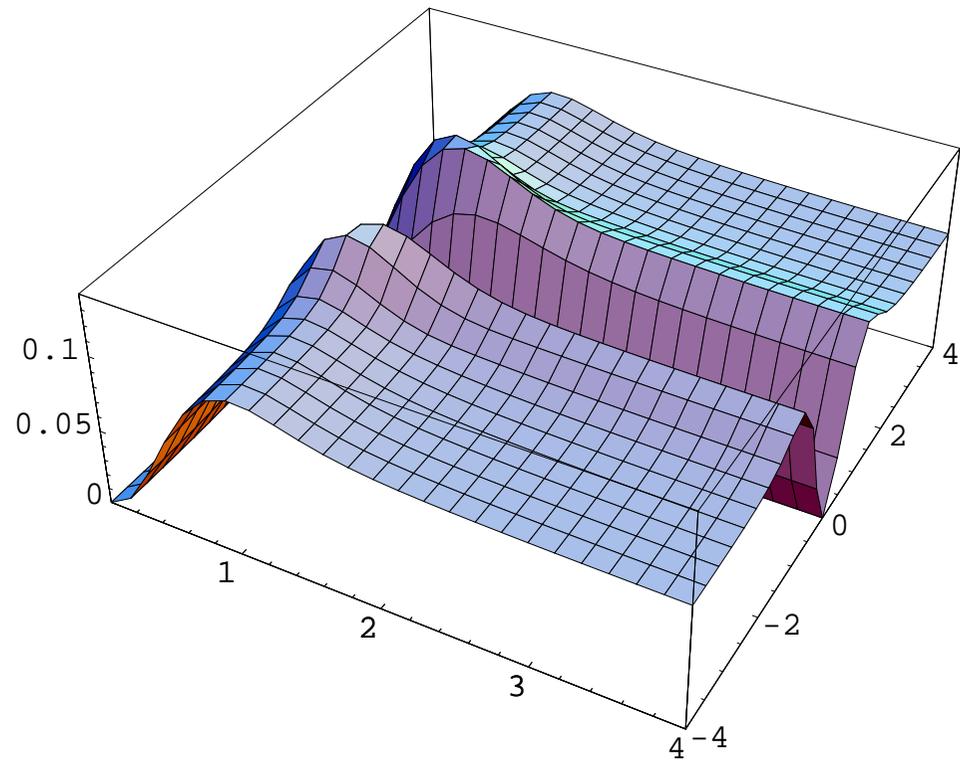
- **quenched density** $\nu = 0$

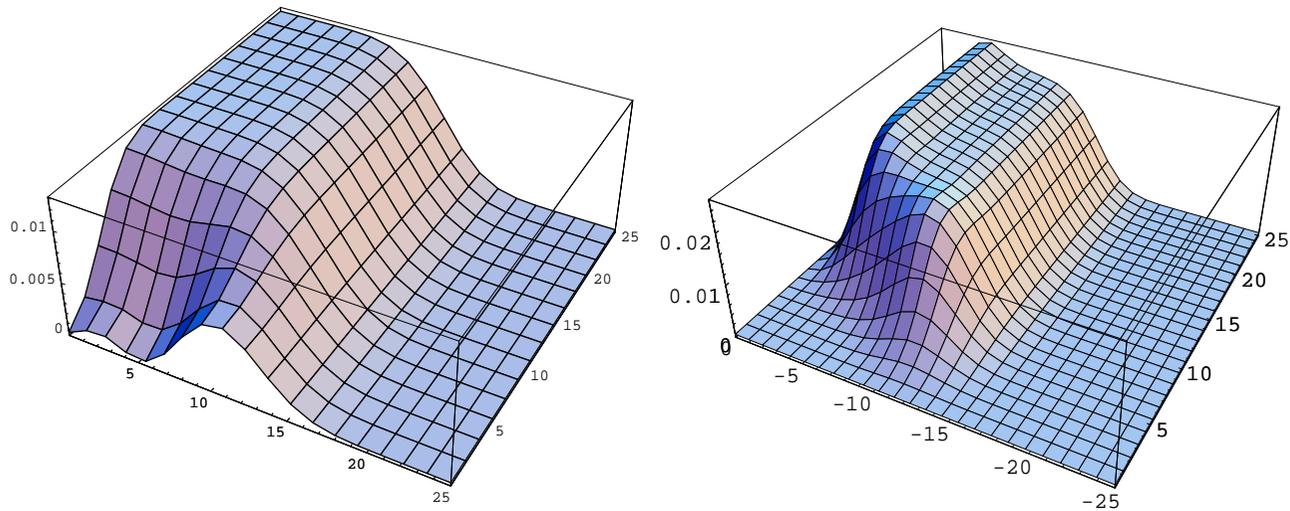• **massive density** $m\sqrt{V}\langle\bar{q}q\rangle \sim 2.5$

- **massive density** $m\sqrt{V}\langle\bar{q}q\rangle \sim 0.5$

• **quenched density** $\nu = 2$
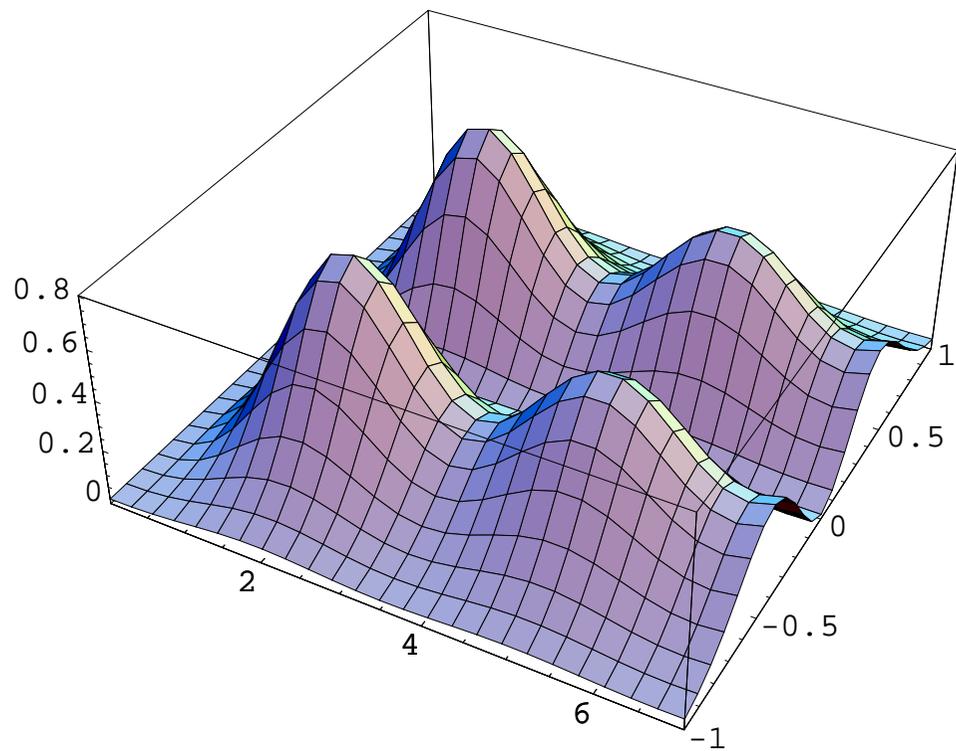
Comparison to unquenched QCD

- pair of $N_f = 2$ $mV\langle \bar{q}q \rangle = 5$, $\mu^2 F^2 V = 6.25$:

  phase quenched QCD(left) vs. SU(2) (right)

- **zero at** $z = im$

- fully **unquenched QCD: oscillations for** $"z > im"$

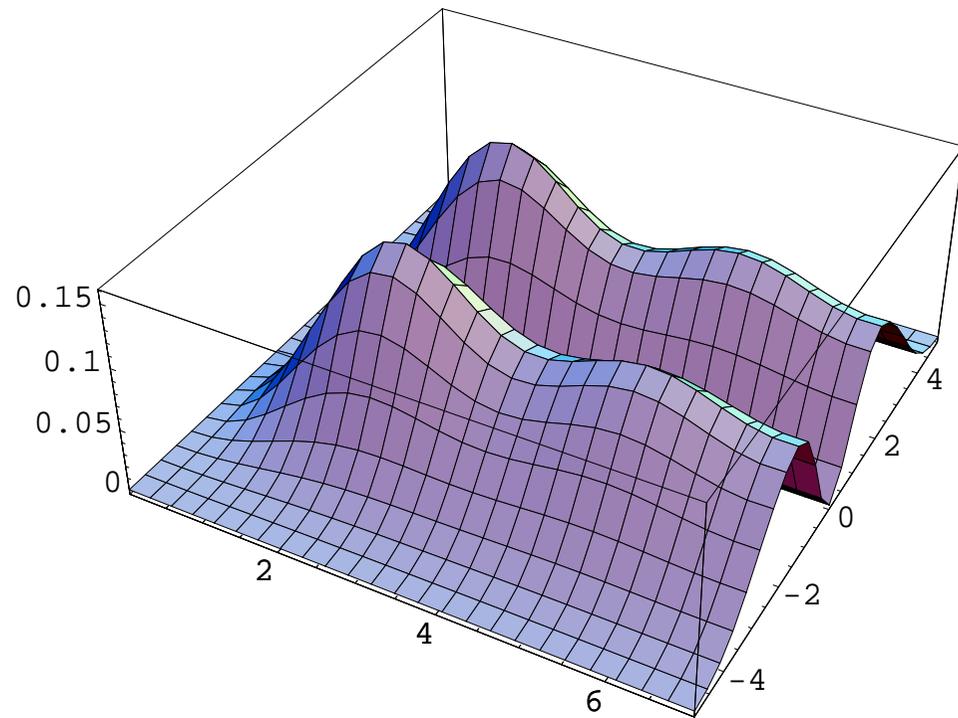 **exponential growth with** $\mu^2 F^2 V + \exists \Im m(\rho) \neq 0$

Summary

- **New results for complex chiral symplectic MM:**

- **describe SU(2) staggered data** quenched & **unquenched!**

- other symmetry class: **real non-symmetric matrices:**

  real + complex eigenvalues at same time, how distributed?

  =   SU(2) / adj. staggered class: **simulations simple !**

- **relation to χPT $+ \mu$ in ε-regime: SU(2)**
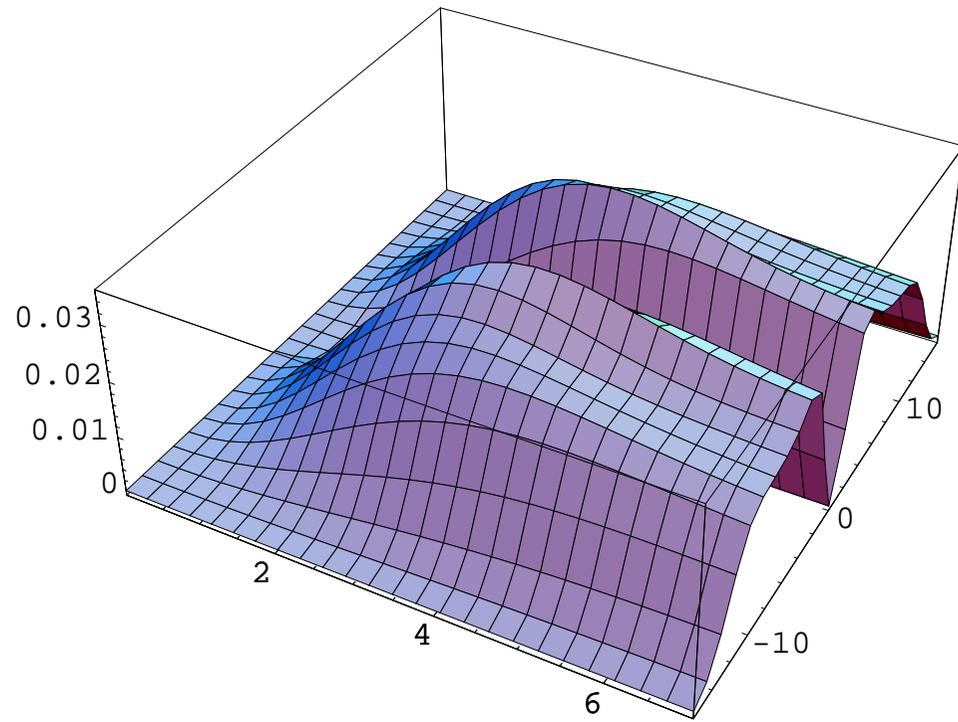
Influence of scaling parameter $\mu^2 F^2 V$

- $\mu^2 F^2 V = 0.4$

- $\mu^2 F^2 V = 1.2$

- $\mu^2 F^2 V = 2.5$